Homework 2

MATH 215 (due Septeber 9) September 7, 2022

Problem 1. Formalize each of the following statements using the propositional calculus.

(a) Every solution of $x^2 - 5x + 6 = 0$ is positive.

(b) Every prime number is greater than 1.

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Problem 2. For each of the following statements, write the negation of the sentences **without** the negation symbol "$\neg$", and prove the negation:

1. $(\exists x. x > 5) \Rightarrow (\forall y. y > -100)$.
2. $\exists \epsilon. (\epsilon > 0) \wedge (\forall x. x > 0 \Rightarrow x > \epsilon)$.
3. $\forall x. (x > 5) \Leftrightarrow (\forall y. y > -100)$.
   (Hint: Recall that $A \Leftrightarrow B \equiv (A \Rightarrow B) \wedge (B \Rightarrow A)$)

Problem 3. Prove the following statement:

For all integers a, b, and for every positive integer n, if both a and b are multiple of n, then $a - b$ is a multiple of n.

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Problem 4. Prove or disprove (prove their negation) the following statements:

a. For all integers n, a, b, c, if $a - b$ and $b - c$ are multiples of n, then $a - c$ is a multiple of n.

b. $\forall x. x^2 < 5 \lor 2x + 1 \geq 7$.

c. $\forall x.(\forall y. y + x < y) \lor (\exists y. 0 < y \land y < x)$

d. For all integer a, b, if both $a + 1$ and $b + 1$ are even, then $ab + 1$ is even.

e. $\forall x. \exists y. x + y > y \Rightarrow x^2 < 0$.

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Problem 5. Prove the following universal implication:

If n is even then $n + 2$ is even.