Homework 11

MATH 215 (due November 25) November 18, 2022

Problem 1. Let $f : A \to B$ and $g : B \to C$ be function. Prove the following items:

1. If f, g are injective then $g \circ f$ is injective.
2. If f, g are surjective, then $g \circ f$.

Problem 2. Prove that the following functions are invertible and find their inverse:

1. $h : (0, \infty) \to (0, 1)$ defined by $h(x) = \frac{1}{1+x^2}$
2. $f : \mathbb{N} \to \mathbb{N}$ defined by $f(n) = \begin{cases} n+1 & n \in \mathbb{N}_{even} \\ n-1 & n \in \mathbb{N}_{odd} \end{cases}$.
3. $g : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ defined by $g(\langle n, m \rangle) = \langle n, n+m \rangle$

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Problem 3. Define

$$f_1 : \mathbb{N} \to \mathbb{N} \times \mathbb{N}, \quad f_1(n) = \langle n+1, n+2 \rangle$$

$$f_2 : \mathbb{N} \to \mathbb{N}, \quad f_2(n) = n^2$$

$$f_3 : \mathbb{N} \times \mathbb{N} \to \mathbb{Z}, \quad f_3(\langle n, m \rangle) = n - m$$

$$f_4 : \mathbb{N} \to \mathbb{N}, \quad f_4(n) = n + 1$$

Determine if the following compositions are defined and compute them:

1. $f_1 \circ f_2$ and $f_2 \circ f_1$.

2. $f_2 \circ f_2$. and $f_3 \circ f_3$

3. $f_4 \circ f_2$ and $f_2 \circ f_4$.

4. $f_3 \circ f_1 \circ f_2$ and $f_4 \circ f_3 \circ f_2$.

Problem 4. Let $A, B \neq \emptyset$ be any set and let $f : A \to B$ be a function. Define a new function using f, as follows, $F : P(A) \to P(B)$ defined by $F(X) = \{f(x) \mid x \in X\}$. Prove that f is invertible if and only if F is invertible.