

Homework 11

MATH 300

(due May 9)

May, 2025

Problem 1. Prove that for any $n \geq 1$, $\mathbb{R}^n \sim \mathbb{R}$. [Hint: by induction on n , you can use the result from class that $\mathbb{R} \times \mathbb{R} \sim \mathbb{R}$.]

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Problem 2. Prove that for every set A , $A \times A$ is an equivalence relation.

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Problem 3. For each of the following relations E over the set A , check whether it is reflexive symmetric or/and transitive. If it is an equivalence relation, determine how many elements are in the A/E :

1. $A = P(\mathbb{R}), E = \{\langle X, Y \rangle \in P(\mathbb{R}) \times P(\mathbb{R}) \mid X \cap Y \neq \emptyset\}$
2. $A = \mathbb{R} \setminus \{0\}, E = \{\langle x, y \rangle \in (\mathbb{R} \setminus \{0\})^2 \mid xy > 0\}$
3. (*optional) $A = {}^{\mathbb{N}}\mathbb{N}, E = \{\langle f, g \rangle \in ({}^{\mathbb{N}}\mathbb{N})^2 \mid \exists N \in \mathbb{N} \forall n \geq N, f(n) = g(n)\}$.

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Problem 4. Prove that if R is a reflexive relation on A if and only if $id_A \subseteq R$.