MATH 300 **Homework 1-Solutions** Feb 2025

Problem 1. The following sentences can be expressed in the form $P \Rightarrow Q$. For each sentence, identify the antecedent (P) and the consequent (Q).

(a) My thumb will hurt if I hit it with a hammer.

(b) The sun is not visible whenever the sky is overcast.

(c) You graduate from college only if you graduate from high school.

(d) Taking the final is necessary for passing the class.

(e) Scoring 90% or above in this class is sufficient for getting A.

(f) Answering this question wrong implies you didn't study hard enough.

Solution. (a) $\underbrace{\text{My thumb will hurt}}_{Q}$ if $\underbrace{\text{I hit it with a hammer}}_{P}$.

(b) $\underbrace{\text{The sun is not visible}}_{Q}$ whenever $\underbrace{\text{the sky is overcast}}_{P}$.

(c) $\underbrace{\text{You graduate from college}}_{P}$ only if $\underbrace{\text{you graduate from high school}}_{Q}$.

(d) $\underbrace{\text{Taking the final}}_{Q}$ is necessary for $\underbrace{\text{passing the class}}_{P}$.

(e) $\underbrace{\text{Scoring 90\% or above in this class}}_{P}$ is sufficient for $\underbrace{\text{getting A}}_{Q}$.

(f) $\underbrace{\text{Answering this question wrong}}_{P}$ implies $\underbrace{\text{you didn't study hard enough}}_{Q}$.

Problem 2. Use truth tables to show the following:

(a) $\neg(P \vee Q) \equiv (\neg P) \wedge (\neg Q)$.

(b) $P \Rightarrow Q \equiv (P \vee Q) \Rightarrow Q$.

(c) $(P \wedge Q) \vee (P \Rightarrow (\neg Q)) \equiv T$.

(d) $(P \vee (\neg Q)) \wedge (P \Rightarrow Q) \not\equiv F$.

Solution. For each point, one should compute the truth tables of the two sentences and check they have the same truth values.

(a)

P	Q	$\neg P$	$\neg Q$	$P \vee Q$	$\neg(P \vee Q)$	$(\neg P) \wedge (\neg Q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

(b)

P	Q	$P \vee Q$	$P \Rightarrow Q$	$(P \vee Q) \Rightarrow Q$
T	T	T	T	T
T	F	T	F	F
F	T	T	T	T
F	F	F	T	T

(c)

P	Q	$\neg Q$	$P \wedge Q$	P $\Rightarrow (\neg Q)$	$(P \wedge Q) \vee (P \Rightarrow (\neg Q))$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	F	T	T

(d) For example $V(P) = F$ and $V(Q) = F$ makes the statement true and so it is not a contradiction.

Problem 3. Which of the statements below are equivalent to Q∨P? (Could be none/more than one.) Show your computation/reasoning.

(a) $\neg((\neg Q) \wedge (\neg P))$

(b) $(\neg Q) \Rightarrow P$

(c) $Q \vee (P \wedge Q)$

(d) $(Q \wedge (\neg P)) \vee P$

Solution. We compute the truth tables of the given sentences.

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

(a)

P	Q	$\neg Q$	$\neg P$	$(\neg Q) \wedge (\neg P)$	$\neg((\neg Q) \wedge (\neg P))$
T	T	F	F	F	T
T	F	T	F	F	T
F	T	F	T	F	T
F	F	T	T	T	F

(b)

P	Q	$\neg Q$	$(\neg Q) \Rightarrow P$
T	T	F	T
T	F	T	T
F	T	F	T
F	F	T	F

(c)

P	Q	$P \wedge Q$	$Q \vee (P \wedge Q)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	F

(d)

P	Q	$\neg P$	$Q \wedge (\neg P)$	$(Q \wedge (\neg P)) \vee P$
T	T	F	F	T
T	F	F	F	T
F	T	T	T	T
F	F	T	F	F

We conclude that $Q \vee P$ is equivalent to (a), (b), and (d). However, it is not equivalent to (c).

Problem 4. Formalize the following statement in the propositional calculus (construct a dictionary, write the premises and conclusions as formulas), and determine weather or not the conclusion logically follows from the premises. Prove your answer.

"Alice said that if Bob or Deshaun will take the test then she wont take the test. Deshaun said that if Cory and Alice will take the test then so will he. Therefore, if Alice and Cory will not take the test, Deshaun will take the test."

solution A ="Alice will take the test, B ="Bob will take the test", C ="Cory will take the test", D ="Deshaun will take the test"

The premises:

(a) $(B \vee D) \Rightarrow \neg A$.

(b) $(C \wedge A) \Rightarrow D$.

The conclusion: $((\neg A) \wedge (\neg C)) \Rightarrow D$

The conclusion does not follow from the premises. For example, consider the true value assignment $V(A) = V(C) = V(B) = V(D) = F$. This truth value assignment makes the premises True but the conclusion false.

Problem 5. For each of the following statements, determine if it is a predicate/statement(no explanation required) and mark all the free variables:

1. $(\exists x(x > 5)) \Rightarrow (\forall y(y > -100))$.
2. $(\epsilon > 0) \wedge (\forall x(x > 0 \Rightarrow x > \epsilon))$.
3. There is a number x which is greater than every other number.
4. For every n, then number $m + n$ has at list two distinct prime divisors.

Solution

(1) Statement.

(2) predicate, ϵ is free x is bounded.

(3) statement.

(4) predicate, m is free, n is bounded.

Problem 6 (Not graded for points)**.** In a fictional village, every inhabitant is either a truth-teller (everything they say is true) or a liar (everything they say is false). Arnie and Barnie live in the village. Suppose that Arnie says, "If I am a truth-teller, then so is Bernie." Are Arnie and Barnie truth-tellers or liars? Motivate your answer.
[Hint: Let A be the statement "Arnie is a truth-teller" and let B be the statement "Barnie is a truth-teller." Arnie's statement can then be expressed as $A \Rightarrow B$. Create a truth table for Arnie's statement...]

Solution: We do not know if A is true of not. If A is true so should $A \Rightarrow B$ and if A is false so should $A \Rightarrow B$. Namely, what should be true is $A \iff (A \Rightarrow B)$. The truth table for this is:

A	B	$A \Rightarrow B$	$A \iff (A \Rightarrow B)$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	F

So the only situation that $A \Leftrightarrow (A \Rightarrow B)$ is true is if both A, B are true.