Mathematical Reasoning-Final Exam

MATH 300 (Instructor: Tom Benhamou) December 18, 2024

Instruction

The final exam consists of 5 problems, each worth 21 points (The maximal grade is 100). The duration of the exam is 3 hours. No external material/equipment is authorized. You can only rely on statements we have seen in class and proof techniques we have presented in class. The answers to the problems should be answered in the designated areas.

Full Name (PRINT): ______________________________

Net ID: ______________________________

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Problems

Problem 1. Let f_n be the sequence defined recursively by: $f_0 = f_1 = 1$ and $f_{n+1} = 2f_n + f_{n-1}$. Prove that for every $n \in \mathbb{N}$, $f_n \leq 3^n$.

Solution 1:

Problem 2. Let $A = \{x \in \mathbb{R} \mid \exists z_1, z_2 \in \mathbb{Z},\ x - z_1 = z_2 - x\}$.

(a) Give an example of an element of A and an example of an element not in A. No proof is required.

(b) Prove that $A \sim \mathbb{N}$.

Solution 2:

Problem 3. Answer the following items, no proof required:

a. Determine whether $(A \Rightarrow (B \Rightarrow \neg C)) \equiv (A \vee B) \Rightarrow C$

is true / is false

b. Determine whether $\forall A \forall B,\ P(A \cap B) = P(A) \cap P(B)$

is true / is false

c. Determine whether $\exists A \exists B,\ P(A \setminus B) = P(A) \setminus P(B)$

is true / is false

d. Consider the statements

$$\alpha_1 = (A \wedge B) \Rightarrow C,\ \alpha_2 = (\neg C) \vee A \text{ and } \alpha = A \wedge B$$

Determine whether the conclusion α:

logically follows from α_1, α_2 / **does not** logically follow from α_1, α_2.

Problem 4. Let $H : \mathbb{N} \setminus \{0,1\} \to P(\mathbb{N})$ be defined by

$$H(n) = \{m \in \mathbb{N} \setminus \{0,1\} \mid n \text{ is divisible by } m\}.$$

(a) What is $H(12)$? no proof is required.

In the following questions, prove your answer.

(b.) Is H one-to-one?

(c.) Is H onto?

Solution 4:

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Problem 5. Let $\sim$ be the equivalence relation on $\{0,1\}^6$:

$$\langle a_1, a_2, a_3, a_4, a_5, a_6\rangle \sim \langle b_1, b_2, b_3, b_4, b_5, b_6\rangle \Leftrightarrow a_1+a_2+a_3+a_4+a_5+a_6 = b_1+b_2+b_3+b_4+b_5+b_6$$

(a) Prove that $\sim$ is an equivalence relation.

In the following questions, no proof is required.

(b.) Compute $[\langle 1,0,0,0,0,0\rangle]_\sim$.

(c.) What is $|\{0,1\}^6/\sim|$?

Solution 5:

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