Mathematical Reasoning-Final Exam

MATH 300 (Instructor: Tom Benhamou) May 5, 2023

Instruction

The final exam consists of 5 problems, each worth 21 points (The maximal grade is 100). The duration of the exam is 3 hours. No external material/equipment is authorized. You can only rely on statements we have seen in class and proof techniques we have presented in class. The answers to the problems should be answered in the designated areas.

Problems

Problem 1. Prove by induction that $2^{2n} - 1$ is divisible by 3, for all integers $n \geq 0$.

Solution 1:

Problem 2. Let

$$\mathbb{B} = \left\{ S \in P(\mathbb{N}) \,\middle|\, \forall n \in \mathbb{N},\ n \in S \to n+1 \in S \right\}.$$

(1) Give an example of two different sets in $\mathbb{B}$, and one set not in $\mathbb{B}$. No proof required.

(2) Prove that $\mathbb{B}$ is infinitely countable.

Solution 2:

Mathematical Reasoning-Final Exam

MATH 300 (Instructor: Tom Benhamou) May 5, 2023

Problem 3. Answer the following items, no proof required:

a. Determine whether $(A \Rightarrow (B \Rightarrow C)) \equiv (A \wedge B) \Rightarrow C$

is true / is false

b. Determine whether $\forall A \forall B,\ P(A \cup B) = P(A) \cup P(B)$

is true / is false

c. Consider the statements

$$\alpha_1 = ((\neg A) \wedge B) \Rightarrow C,\ \alpha_2 = (\neg C) \vee A \text{ and } \alpha = A \vee \neg B$$

Determine whether the conclusion α:

logically follows from α_1, α_2 / **does not** logically follow from α_1, α_2.

Mathematical Reasoning-Final Exam

MATH 300 (Instructor: Tom Benhamou) May 5, 2023

Problem 4. Let $f, g : \mathbb{R} \to P(\mathbb{N})$ be any functions and $h : \mathbb{R} \times \mathbb{R} \to P(\mathbb{Z})$ be defined by $h(n, m) = f(n) \cup -g(m)$ where $-g(m) = \{-x \mid x \in g(m)\}$.

(a) Prove or give a counterexample (without proving) for f, g, h: if f, g are one-to-one then h is one-to-one.

(b) Prove or give a counterexample (without proving) for f, g, h: if f, g are onto $P(\mathbb{N})$ then h is onto $P(\mathbb{Z})$.

Solution 4:

Mathematical Reasoning-Final Exam

MATH 300 (Instructor: Tom Benhamou) May 5, 2023

Problem 5. Let $E = \{\langle A, B\rangle \in P(\mathbb{Z})^2 \mid A\Delta B \text{ is finite}\}$.

1. Prove that $A\Delta C \subseteq A\Delta B \cup B\Delta C$.

2. Prove that E is an equivalence relation on $P(\mathbb{Z})$.

Solution 5: