MATH 504 PROBLEM SET 8

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Problem 1. The following enumeration refers to the enumeration in the exercises section of chapter VII in Kunen's book "Set Theory: An Introduction to Independence Proofs":
$(A5), (A7), (A8), (B5), (B6), (B13)$

Problem 2. Suppose that P, Q are two posets in the ground model M and $\pi : P \to Q$ is a projection, i.e.
(a) π is order-preserving: $p' \leq p \to \pi(p') \leq \pi(p)$.
(b) for all $p \in P$ and $q \leq \pi(p)$, there is $p' \leq p$, such that $\pi(p') \leq q$.
Suppose that G is a P-generic filer over V. Show that

$$H := \{q \mid \exists p \in G, \pi(p) \leq q\}$$

is a Q-generic filter over V.

Problem 3. In several arguments, we will reach the conclusion that some condition $p \in \mathbb{P}$ forces some contradiction. Prove that if ϕ is a contradiction then there is no p such that $p \Vdash \phi$. [Remark: this is a one-line proof.]

(1) Let $\mathbb{P} = Add(\omega, 1)$ be the Cohen forcing. Let G be M-generic for $\mathbb{P}$. Show that for any $A \in M[G]$, $A \subseteq (\omega_1)^M$, there is $B \in M$, $|B| = \omega_1$ such that either BA or $B \subseteq \omega_1 \setminus A$.

[Hint: Let $\underset{\sim}{A}$ be a name for A. In M, let

$$D = \{p \in \mathbb{P} \mid \exists B, |B| = \omega_1 \wedge [p \Vdash \hat{B} \subseteq \underset{\sim}{A}] \vee [p \Vdash \hat{B} \subseteq \hat{\omega}_1 \setminus \underset{\sim}{A}]\}$$

It suffices to prove that D is dense. Let $p \in Add(\omega, 1)$ be any condition, for each $\alpha < \omega_1$, find $p_\alpha \leq p$ such that $p_\alpha || \hat{\alpha} \in \underset{\sim}{A}$. Use the pigeonhole principle to find now a single $p^* \leq p$ which decides $\hat{\alpha} \in \underset{\sim}{A}$ for ω_1-many α's.]