What the Tortoise Said to Achilles

Lewis Carroll

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Achilles had overtaken the Tortoise, and had seated himself comfortably on its back.

"So you've got to the end of our race-course?" said the Tortoise. "Even though it does consist of an infinite series of distances? I thought some wiseacre or other had proved that the thing couldn't be done?"

"It *can* be done," said Achilles. "It *has* been done! *Solvitur ambulando*. You see the distances were constantly *diminishing*; and so --"

"But if they had been constantly *increasing*?" the Tortoise interrupted "How then?"

"Then I shouldn't be *here*," Achilles modestly replied; "and you would have got several times round the world, by this time!"

"You flatter me -- *flatten*, I *mean*" said the Tortoise; "for you are a heavy weight, and *no* mistake! Well now, would you like to hear of a race-course, that most people fancy they can get to the end of in two or three steps, while it *really* consists of an infinite number of distances, each one longer than the previous one?"

"Very much indeed!" said the Grecian warrior, as he drew from his helmet (few Grecian warriors possessed *pockets* in those days) an enormous note-book and a pencil. "Proceed! And speak *slowly*, please! *Shorthand* isn't invented yet!"

"That beautiful First Proposition of Euclid!" the Tortoise murmured dreamily. "You admire Euclid?"

"Passionately! So far, at least, as one *can* admire a treatise that won't he published for some centuries to come!"

"Well, now, let's take a little bit of the argument in that First Proposition -- just *two* steps, and the conclusion drawn from them. Kindly enter them in your notebook. And in order to refer to them conveniently, let's call them *A*, *B*, and *Z*: --

(*A*) Things that are equal to the same are equal to each other.

(*B*) The two sides of this Triangle are things that are equal to the same.

(*Z*) The two sides of this Triangle are equal to each other.

Readers of Euclid will grant, I suppose, that *Z* follows logically from *A* and *B*, so that any one who accepts *A* and *B* as true, *must* accept *Z* as true?"

"Undoubtedly! The youngest child in a High School -- as soon as High Schools are invented, which will not be till some two thousand years later -- will grant *that.*"

"And if some reader had *not* yet accepted *A* and *B* as true, he might still accept the *sequence* as a *valid* one, I suppose?"

"No doubt such a reader might exist. He might say 'I accept as true the Hypothetical Proposition that, *if A* and *B* be true, *Z* must be true; but, I *don't* accept *A* and *B* as true.' Such a reader would do wisely in abandoning Euclid, and taking to football."

"And might there not *also* he some reader who would say 'I accept *A* and *B* as true, but I *don't* accept the Hypothetical '?"

"Certainly there might. *He*, also, had better take to football."

"And *neither* of these readers," the Tortoise continued, "is *as yet* under any logical necessity to accept *Z* as true?"

"Quite so," Achilles assented.

"Well, now, I want you to consider *me* as a reader of the *second* kind, and to force me, logically, to accept *Z* as true."

"A tortoise playing football would be -- " Achilles was beginning

"-- an anomaly, of course," the Tortoise hastily interrupted.

"Don't wander from the point. Let's have *Z* first, and football afterwards!"

"I'm to force you to accept *Z*, am I?" Achilles said musingly. "And your present position is that you accept *A* and *B*, but you don't accept the Hypothetical --"

"Let's call it *C*," said the Tortoise.

"-- but you *don't* accept

(*C*) If *A* and *B* are true, *Z* must be true. "

"That is my present position," said the Tortoise.

"Then I must ask you to accept *C*."

"I'll do so," said the Tortoise, "as soon as you've entered it in that note-book of yours. What else have you got in it?"

"Only a few memoranda," said Achilles, nervously fluttering the leaves: "a few memoranda of -- of the battles in which I have distinguished myself!"

"Plenty of blank leaves, I see!" the Tortoise cheerily remarked. "We shall need them *all*!" (Achilles shuddered.) "Now write as I dictate: --

(*A*) Things that arc equal to the same are equal to each other.

(*B*) The two sides of this Triangle are things that are equal to the same.

(*C*) If *A* and *B* are true, *Z* must be true.

(*Z*) The two sides of this Triangle are equal to each other."

"You should call it *D*, not *Z*," said Achilles. "It comes *next* to the other three. If you accept *A* and *B* and *C*, you *must* accept *Z*."

"And why *must* I?"

"Because it follows *logically* from them. If *A* and *B* and *C* are true, *Z must* be true. You don't dispute *that*, I imagine?"

"If *A* and *B* and *C* are true, *Z must* he true," the Tortoise thoughtfully repeated. "That's *another* Hypothetical, isn't it? And, if I failed to see its truth, I might accept *A* and *B* and *C*',

and *still* not accept *Z*. mightn't I?"

"You might," the candid hero admitted; "though such obtuseness would certainly be phenomenal. Still, the event is *possible*. So I must ask you to grant *one* more Hypothetical."

"Very good. I'm quite willing to grant it, as soon as you've written it down. We will call it

(*D*) If *A* and *B* and *C* are true, *Z* must be true.

"Have you entered that in your notebook?"

"I *have*!" Achilles joyfully exclaimed, as he ran the pencil into its sheath. "And at last we've got to the end of this ideal race-course! Now that you accept *A* and *B* and *C* and *D*, *of course* you accept *Z*."

"Do I?" said the Tortoise innocently. "Let's make that quite clear. I accept *A* and *B* and *C* and *D*. Suppose I *still* refused to accept *Z*?"

"Then Logic would *force* you to do it!" Achilles triumphantly replied. "Logic would tell you 'You can't help yourself. Now that you've accepted *A* and *B* and *C* and *D*, you *must* accept *Z*!' So you've no choice, you see."

"Whatever Logic is good enough to tell me is worth *writing down*," said the Tortoise. "So enter it in your book, please. We will call it

(*E*) If *A* and *B* and *C* and *D* are true, *Z* must be true. Until I've granted *that*, of course I needn't grant *Z*. So it's quite a *necessary* step, you see?"

"I see," said Achilles; and there was a touch of sadness in his tone.

Here narrator, having pressing business at the Bank, was obliged to leave the happy pair, and did not again pass the spot until some months afterwards. When he did so, Achilles was still seated on the back of the much-enduring Tortoise, and was writing in his note-book, which appeared to be nearly full. The Tortoise was saying, "Have you got that last step written down? Unless I've lost count, that makes a thousand and one. There are several millions more to come. And *would* you mind, as a personal favour, considering what a lot of instruction this colloquy of ours will provide for the Logicians of the Nineteenth Century -- *would* you mind adopting a pun that my cousin the

Mock-Turtle will then make, and allowing yourself to be re-named *Taught-Us*?"

"As you please!" replied the weary warrior, in the hollow tones of despair, as he buried his face in his hands. "Provided that *you*, for *your* part, will adopt a pun the Mock-Turtle never made, and allow yourself to be re-named *A Kill-Ease*!"

Transcribed into hypertext by Andrew Chrucky, July 10, 1997.
