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TITLE OF PROPOSED PROJECT **Nonsmooth methods in optimal control theory**

REQUESTED AMOUNT	PROPOSED DURATION (1-60 MONTHS)	REQUESTED STARTING DATE	SHOW RELATED PRELIMINARY PROPOSAL NO. IF APPLICABLE
$ 184,047	36 months	06/01/04	

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PI/PD DEPARTMENT **Mathematics**	PI/PD POSTAL ADDRESS **Department of Mathematics** **Louisiana State University** **Baton Rouge, LA 70803** **United States**
PI/PD FAX NUMBER **225-578-4276**	

NAMES (TYPED)	High Degree	Yr of Degree	Telephone Number	Electronic Mail Address
PI/PD NAME **Peter R Wolenski**	PhD	1988	225-578-1606	wolenski@math.lsu.edu
CO-PI/PD				
CO-PI/PD				
CO-PI/PD				
CO-PI/PD				

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CERTIFICATION PAGE

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NAME **James L Bates**	**Electronic Signature**	**Oct 30 2003 4:45PM**

TELEPHONE NUMBER	ELECTRONIC MAIL ADDRESS	FAX NUMBER
225-578-3386	**osp@lsu.edu**	**225-578-5403**

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Summary of Proposed Research

This proposal seeks funding to carry out fundamental research in the area of optimal control theory, and its methodology is based on nonsmooth and variational analysis. Although we may not always be prescient of its existence, control theory is ubiquitous in our daily lives. There is the presence of "automatic" controllers that are imbedded in engineering design, as for example in DC motors, RC circuits, robot control, chemical processes, automatic flight controllers, etc., and there are direct control processes manifested through human intervention, such as driving an automobile, landing an airplane, planning a harvesting effort, etc. Control theory is eminently placed in engineering science, but since applications tend to outpace the development of theoretical foundations, there are many mathematical issues that crave further rigorous scrutiny. The intellectual merit of this proposal lies in its approach of rigorously analyzing mathematical problems that arise naturally in applied control applications. Science abounds with models describing continuous movement of some sort, and differential equations are employed for this purpose. As science evolves into technology, and human concerns and influences become paramount, then certain outcomes are preferred over others and more efficient movements are desired. Hence issues of optimization are injected into the model, and *dynamic* optimization problems arise. This is *optimal* control theory. Although optimal control is not so prominent in control engineering, it nonetheless has a very high mathematical content with historic roots grounded in the calculus of variations. It straddles the boundary between pure and applied mathematics, where on the practical side, it offers the applied engineer insights, philosophical approaches, and some of the mathematical tools needed to attack effectively practical problems. However, many control applications do not fit neatly into a classical theoretical setting, where differentiability is often a prerequisite. Thus theoretical challenges emerge to provide the requisite foundation, and the mathematical theory of nonsmooth and variational analysis has been developed over the previous three decades largely motivated by these practical control and optimization problems. This theory now consists of a substantial and complete body of results, and currently is seeking further applications. Indeed, it is being increasingly appreciated and utilized by engineers. Just as the world appears to be truly nonlinear, the world also seems to have many more nonsmooth characteristics than previously thought. The broad impact of this proposal is that further nonsmooth tools will become accessible to the control community, and this in turn will embolden control engineers to devise more realistic mathematical models.

More specifically, this proposal takes aim at several problem formulations where the mathematics is not adequately developed. The Fully Convex Control problem is a generalization of the Linear Quadratic Regulator, the well-known workhorse in control, but allows for realistic constraints that are usually discarded for the sake of simplicity. State constrained, impulsive, and infinite horizon problems will be studied in this framework. Further research problems are described for One-Sided Lipschitz, time-delay, and nonlinear impulsive problems, where recent theoretical results obtained in the standard problem have yet to be extended. In particular, the new One-Sided Lipschitz theory could lead to a maximum principle that would apply to models with dry friction.

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1 Results from previous NSF proposals

The PI has been awarded two previous NSF proposals, DMS-9623406 and DMS-9972241; however, the duration of each was for only two years. There was no attempt last year to renew this funding because the goals of the previous proposal had not yet been achieved, and the case for continuing support at that time would probably not have been competitive. However, over the last year, not only have many of those goals come to fruition, but new directions have emerged to significantly expand his proposed research program. It is hoped that the present proposal justifies a three year award so that the goals can be obtained under the time-frame of the actual funding.

We shall assume the reviewer is somewhat familiar with the basic concepts of nonsmooth and variational analysis [13, 42].

1.1 Fully convex problems

The main results from NSF proposal DMS-9623406 were in so-called "Fully Convex Control" (FCC) problems, and the papers [43, 44] co-authored with R.T. Rockafellar were the main published outcomes. This research also directly influenced the research direction of Rockafellar's last two Ph.D. students Grant Galbraith and Rafal Goebel. The PI is presently working with Goebel on extensions and variants of this topic, and this new research direction will be described below in §2.4.

The theory of FCC problems is centered around studying a variational problem of form

$$P(\tau,\xi) \qquad\qquad \min \ell\big(x(0)\big) \;+\; \int_0^\tau L\big(x(t),\dot{x}(t)\big)\,dt$$

subject to the terminal conditions $x(\tau)=\xi$. The *Lagrangian* $L : I\!R^n \times I\!R^n \to (-\infty,\infty]$ is extended-valued to incorporate constraints (a now standard technique in variational analysis) and is assumed to be *jointly* convex in (x,v). It is also assumed that $\ell(\cdot) : I\!R^n \to (-\infty,\infty]$ is convex. The convexity assumptions make the problem somewhat special. On the other hand, FCC is an important class of problems that roughly resembles the relationship to general nonlinear control theory that linear analysis plays in classical functional analysis. Strong conclusions that are global and include complete subgradient characterizations can be drawn in this framework, whereas more generally, typically only local and partial inclusions are available. But FCC also includes the important subclass of Linear Quadratic Control, which is so prominent in applications, and also less well-known extensions that incorporate control constraints. The Lagrangian also satisfies additional technical assumptions that need not be stated here.

One main result of [43] reveals that the optimal value function $V(\tau, \xi)$ is lower semicontinuous and satisfies the generalized Hamilton-Jacobi equation

$$(\sigma, -\eta) \in \partial V(\tau, \xi) \quad \Longleftrightarrow \quad \sigma = H(\xi, \eta). \tag{1}$$

The subgradient is in the so-called regular ([42]) or Dini ([13]) sense, and the *Hamiltonian* $H : I\!R^n \times I\!R^n \to I\!R$ is given by

$$H(\xi, \eta) = \sup_{v \in R^n} \big\{ \langle \eta, v \rangle - L(\xi, v) \big\}. \tag{2}$$

We remark that the important uniqueness issue of whether the value function is characterized as this solution (plus appropriate boundary conditions) is not covered by the usual viscosity theory [3, 19], and was proved by Galbraith [27] using invariance and optimization techniques. The somewhat remarkable and distinguishing feature of FCC is that (1) is *if and only if*, whereas typically, only the ($\Rightarrow$) direction holds. Also $+\infty$ values are admissible that correspond to infeasibilities.

The convexity assumptions on L and $\ell(\cdot)$ imply the convexity of $V(\tau, \cdot)$ for each $\tau > 0$. The Legendre-Fenchel conjugate of $V(\tau, \cdot)$ is shown [43] to be a value function of an FCC problem of the same form (with reversed time), and its optimal arcs turn out to be the multipliers (or dual arcs) of the original problem ($P(\tau, \xi)$). Moreover, the pair of optimal/multiplier arcs $\big(x(\cdot), y(\cdot)\big)$ satisfy the Hamiltonian inclusion

$$\big(-\dot{y}(t), \dot{x}(t)\big) \in \partial H\big(x(t), y(t)\big) \quad \text{a.e. } t \in [0, \tau], \tag{3}$$

and the transversality condition $y(0) \in \partial \ell(x(0))$, and these solutions are precisely the *global* characteristics of (1). That is, $(\sigma, \eta) \in \partial V(\tau, \xi)$ *if and only if* there exists a solution $\big(x(\cdot), y(\cdot)\big)$ of (3) with $y(\tau) = -\eta$, $x(\tau) = \xi$, $y(0) \in \partial \ell(x(0))$, and $\sigma = H(\xi, \eta)$. Again, note the global and if and only if character of these results that are deducible only in the FCC framework.

The results in [44] show that the HJ equation (1) is not the only way to get at the value function, and that the envelope formula

$$V(\tau, \xi) = \sup_{\eta \in R^n} \big\{ K(\tau, \xi, \eta) - \ell^*(-\eta) \big\} \tag{4}$$

holds, where $\ell^*(\cdot)$ is the Legendre-Fenchel conjugate of $\ell(\cdot)$ and K is the so-called dualizing kernel. The surprising properties of K are explored in [44]. There is an interesting interpretation of (4) via the max-plus algebra theory, where K can be viewed as a dual object to a Green's function that is admitting an "integral" formula for a solution to (1). Also, (4) is a generalization of the well-known Hopf-Lax formulas that hold when L is independent of x.

1.2 Subgradient Formulas for minimal time functions

Funds from DMS-9972241 also provided partial support for the PI to travel to Padova Italy to collaborate with Giovanni Colombo. We briefly explain the material of the resulting papers [16, 17, 18]. The continuation of research in this direction will be described in §2.5 below.

The goal is to describe the subgradients of a particularly simple class of minimal time functions in terms of the given data, and to derive regularity properties from these formulas. Suppose X is a Hilbert space and $S \subseteq X$ is closed and $F \subseteq X$ is closed, bounded, convex, and $0 \in \text{int } F$. The minimal time function is defined by

$$T_S(x) = \inf\big\{t : \{x + tF\} \cap S \neq \emptyset\big\}. \tag{5}$$

It is a particularly simple example of a *minimal time function* from optimal control theory in which the dynamic data F is a constant set. If $F = \bar{B}$, the closed unit ball in X, then $T_S(x)$ reduces to the distance function $d_S(x)$ to S, which has been extensively studied over the years by many authors. Our earlier paper [14] (see also [39]) characterized those sets S around which $d_S(\cdot)$ was differentiable, and our goal here was to find conditions on S and F for which similar properties would hold. The key result is the following description of the subgradient of $T_S(\cdot)$:

$$\partial T_S(x) = N_{S(r)}(x)(\cap \{\zeta : \rho_{F^\circ}(-\zeta) = 1\},$$

where ρ_{F° is the gauge function of the polar F° of F, and the subgradient and normal cone are either both in the proximal or Frechet/contingent sense. The set $S(r)$ is the r-level set of $T_S(\cdot)$, and $T_S(x) = r$ in this description. Conditions are explored in [16, 17] which lead to regularity properties (C^1 and semiconvexity) of $T_S(\cdot)$.

The broader interest of these results in applications is the understanding of feedback laws in control engineering, since these are generally constructed from the knowledge of the gradient of a Lyapunov function. Our results illustrate the interplay between the geometries of the target set S and the dynamic velocity set F.

1.3 Strong Invariance and One-Sided Lipschitz maps

Travel funds from DMS-9972241 brought the PI into contact with Elza Farkhi, who introduced him to the concept of a One-Sided Lipschitz (OSL) assumption. We subsequently wrote the paper [26]. The research is continuing in collaboration with Tzanko Donchev and my current graduate student Vinicio Rios. We wrote [40], and these are followed up by [23, 24, 25]. Proposed research in OSL will be further described in §2.1 below, and here we give a brief summary of some of our results (only global descriptions are given for simplicity of exposition; the published results have local versions).

A multifunction $F : I\!\!R \rightrightarrows I\!\!R^n$ is *Lipschitz* (in the Hausdorf metric) provided there exists $k > 0$ so that

$$\big|H(x,p) - H(y,p)\big| \leq k\big|p\big|\,\big|x-y\big| \tag{6}$$

for all x, y, and p in $I\!\!R^n$. Again, H is the maximized Hamiltonian as in (2) where the Lagrangian $L(x,\cdot)$ is defined as the indicator of $F(x)$. More explicitly,

$$H(x,p) = \sup_{v \in F(x)} \langle v, p\rangle. \tag{7}$$

Definition (6) is equivalent to the usual definition involving two set inclusions, but can be more transparently compared to the property that F is One-Sided Lipschitz (OSL); the latter requires the existence of $k > 0$ so that

$$H(x, x-y) - H(y, x-y) \leq k\,\big|x-y\big|^2 \tag{8}$$

holds for all x and y. Clearly (6) implies (8), but (8) is strictly weaker, as can be seen by considering $F(x) = \{\sqrt[3]{-x}\}$ in dimension one. The extreme case where $k = 0$ is when the multifunction F is dissipative (i.e. $-F$ is monotone [42]), a case that has been extensively studied in the PDE literature (and for other reasons in optimization). Another special case is where

$$F(x) = D(x) + G(x) \tag{9}$$

where $D(\cdot)$ is dissipative and $G(\cdot)$ is Lipschitz, and [40] provides a characterization of strong invariance (see [13]) for this case; [23] contains a nonautonomous version. More details and future research directions are described below in §2.1.

1.4 Time Delay variational problems

Another project that was proposed in both previous proposals was to extend the Clarke decoupling principle to time delay systems. This has now been fully accomplished in joint work with my graduate student Norma Ortiz [35, 36], and has led further to the study of neutral variational problems [37, 38]. There is still much work to be done in this area as well, and future research is described in §2.2 below.

2 Proposed research

The proposed research is broken into the following five research topics: (1) theory of One-Sided Lipschitz (OSL) multifunctions, (2) variational problems with time-delay, (3) impulsive systems, (4) Fully Convex Control (FCC) extensions and variants, and (5) subgradient formulas and regularity of value functions.

2.1 One-Sided Lipschitz (OSL) multifunctions

Consider the differential inclusion

$$(DI) \qquad \begin{cases} \dot{x}(t) \in F\big(x(t)\big) & \text{a.e. } t \in [0,T] \\ x(0) = x_0, \end{cases}$$

and suppose $S \subseteq I\!\!R^n$ is closed. Differential inclusions are widely studied [2, 13, 48] as a mathematical model for state-based control systems, and there is a well-known theory grounded in the concepts and methods of nonsmooth and variational analysis, see [13, 48]. Basic hypotheses typically include the upper semicontinuity (or outer continuity [42]) and local boundedness of F, and that each value $F(x)$ is nonempty, closed, and convex. The Lipschitz assumption (6) is also invoked at times when stronger conclusions are sought, such as characterizations of the reachable set, strong invariance, and necessary conditions in optimal control.

An immediate and continuing goal of this proposal is to develop a complete theory for OSL multifunctions (see (1.3) above), which has important applications for modeling dry friction and other mechanical systems that can have sudden "downward" shifts in the admissible velocity sets. As a simple motivation for this theory, consider the problem of finding necessary conditions for controlling the movement of a book on a desktop. The friction force impedes any movement until a significantly large control force is applied, and then "jerks" once a threshold is reached. The dependence of this force is not Lipschitz (or even continuous) on the state variable, and to our knowledge, there is no maximum principle available for this type of data. Although a new theory is being developed by Hector Sussmann [47] that applies to not necessarily Lipschitz data, it apparently does not cover the case of OSL. Nevertheless, our approach is quite different, and is based on the proximal theory of nonsmooth analysis [13] that has been successfully applied elsewhere.

Modeling friction is a subject of active research, and has obvious important applications. However, dry (or Coulomb) friction is difficult to model accurately, and is often described (see e.g. [20], p. 193) by the equation

$$\ddot{x}(t) = g(\dot{x}) - \mu \operatorname{sgn}(\dot{x}) + f(x) + g(t). \tag{10}$$

The term $\mu \operatorname{sgn}(\dot{x})$ is discontinuous, and renders the equation outside classical ODE theory. One approach is to consider $\operatorname{sgn}(\cdot)$ as the multivalued map

$$\operatorname{sgn}(v) = \begin{cases} 1 & \text{if } v > 0 \\ [-1, 1] & \text{if } v = 0 \\ -1 & \text{if } v < 0, \end{cases}$$

and treat (10) as a differential inclusion (after performing the usual device of introducing a new state variable as the velocity). If one now adds a control variable into the equation, since the velocity set is not Lipschitz (or even continuous) in the state component $\dot{x}$, the extant versions of the maximum principle do not apply. However, there is a structural "dissipative" character to the equation (10), and in fact it has the form (9) mentioned earlier. Thus we propose to prove a maximum principle for OSL data, and there is evidence that the recent state-of-the-art necessary conditions in dynamic optimization of F.H. Clarke [10] can be extended to OSL systems. We briefly describe some further technical details.

Clarke's new monograph [10] on necessary conditions in dynamic optimization is, in my view, the most important work in dynamic optimization in the past ten years. It contains methods of wide applicability, and moreover is the culmination of a decade of results by many authors. One of these experts have related their opinion to me that these results are "definitve" in the sense that they are what everyone has been looking for. In any case, one of the key components invoked in the proof in [10] is the so-called "Mordukhovich criterian" (see [42, 13]) that says the Lipschitz property (actually a pseudo-Lipschitz property that in some sense localizes the data) is equivalent to

$$w \in DF(x,v)(y) \quad \Rightarrow \quad |w| \leq k|y|, \tag{11}$$

where DF is the coderivative of F. Recall that for $v \in F(x)$, the coderivative multifunction $DF(x,v)(\cdot)$ is described by $w \in DF(x,v)(y)$ if and only if $(-w,y) \in N^P_{\mathrm{gr}\ F}(x,v)$, where $N^P_{\mathrm{gr}\ F}(x,v)$ denotes the proximal normal cone (see [13]) of the graph gr F of F at (x,v). The constant k in (11) is the Lipschitz constant, and gives a crucial handle to pass to the limit of the "forerunner" of the adjoint arcs obtained from decoupling. The "definitiveness" remark is based on that it seems impossible to say much more, except, however, perhaps one can pass to the limit in a slightly different manner, which is what we propose here. The idea is similar as in ODE theory, where typically a linear growth hypothesis

$$|f(x)| \leq c(1+|x|) \tag{12}$$

is assumed to preclude any potential finite time blow-up; that is, (12) is invoked to bound the absolute value of a solution $x(t)$ to $\dot{x}(t) = f(x(t))$ by $(|x_0| + ct) + c\int_0^t |x(t')\,dt')|$. Then one can apply Gronwall's inequality to get a bound on $|x(t)|$. But the above reasoning can be applied to bound $|x(t)|^2$ under the weaker assumption

$$|\langle f(x), x\rangle| \leq c(1+|x|^2).$$

(i.e. $\left|\frac{d}{dt}|x(t)|^2\right| = 2|\langle \dot{x}(t), x(t)\rangle| \leq 2c(1+|x(t)|^2)$, etc). A major step of this proposed research is to show that a variant of the Mordukhovich criterian (11) characterizes the OSL property (8). We reveal our claim that (8) is equivalent to

$$w \in DF(x,v)(y) \quad \Rightarrow \quad |\langle w, y\rangle| \leq k|y|^2. \tag{13}$$

This is not yet proven and to do so will require some new ideas, however the main prototypical examples of OSL multifunctions satisfy it. Once (13) is proven, then the road to necessary conditions for an OSL multifunction lies open (although it will still be lengthy and full of potential pitfalls). The reasoning behind governing the passing to the limit in [10] is analogous to that sketched above in showing the non-blow up of ODE solutions.

As described in §1.3, we have already obtained results in invariance theory for OSL multifunctions that were previously thought to hold only under Lipschitz assumptions. We next plan to develop a Hamilton-Jacobi theory for OSL systems. Preliminary results [25] on the minimal time function have been written out, but more needs to be done here. The role of weak and strong invariance is well understood in HJ theory [13] (since HJ inequalities characterize these notions), and in view of our recent new results on strong invariance [23, 24], further progress seems likely.

We also are working on infinite dimensional versions, and plan to extend the results of Clarke, Ledyaev, and Radulescu [11] to the OSL setting. There is already substantial progress in this direction which will open potential applications to PDEs.

2.2 Variational problems with time delay

Time delays of one form or another are present throughout most mechanical and electrical systems, but are often ignored in the modeling process for technical simplicity. Often this has no major effect on the engineering design; however, there are many situations where delays play a crucial role, and others where it may be desirable to *use* the time delay as a device for increasing stability (cf. [29]). The mathematical difficulty in treating delays is that the problem essentially becomes infinite dimensional, and finding appropriate compactness conditions and convergence schemes becomes necessarily more complicated.

As mentioned above in §1.4, we have made some progress toward treating time-delay problems with the new tools of nonsmooth analysis that are prevalent throughout this proposal. The joint paper [36] is the first step to modernizing extant necessary conditions (e.g. see[15] and references therein) to incorporate the recent advances of [10], but there are still many details and technical issues to resolve. The main issue remaining is determining precisely the right assumptions to make the limiting arguments work.

The previous paragraph refers only to problems where the delay is in the state argument, and the theory there can proceed by and large along the lines of the undelayed case. It is another story altogether when delays appear in the velocities or the control variables (usually called *neutral* problems). Only recently, have Mordukhovich and Wang [32] proven for the first time necessary conditions for a class of differential inclusions with delays in the velocity variable. Even the existence theory here can be troublesome, but we tackled that issue in [37] for the generalized Bolza problem. We have sketched the decoupling technique in this setting as well, although the details are not yet published, and so we have hope and plans to derive necessary conditions for these class of problems (which will include the main results of [32]).

It is still not entirely clear if our methods will apply to problems with delays in the control variables, although it seems unlikely that they will. Such problems are very challenging and cause major problems in existence theory [45, 49] and generic well-posedness [30]. The wide-spread technique of "infinite penalization" that equivalently embeds optimal control problems into a nonclassical calculus of variations framework does not work with such problems, and so it is not clear how one can proceed using our methods.

Another broad research area that nonsmooth methods have recently been successfully applied is stabilization. I have closely been following the ISS (Input-to-State Stability) developments by Sontag and his school, and plan to extend these Lyapunov approaches to time-delay problems. This is not a major theme of this proposal, but is nonetheless an auxiliary research direction in which much work needs to be done. Nonsmooth methods have had a major impact on nondelay systems theory; for example, [12] resolved a major open issue regarding the equivalence of asymptotic stability and the existence of feedback, and has been influential for the acceptance of discontinuous feedback by control engineers. The nonsmooth groundwork seems to be in place to tackle stability issues of time-delays, which is an active area of engineering research [29] in which nonsmooth methods have yet to play much of a role.

2.3 Impulsive control and variational systems

Impulsive systems arise in a variety of applications where states can move at different time scales. The "slow" movement can be thought of as the usual time progression infinitesimally incremented by dt, and the "fast" movement occurs in a small interval that resembles the effect of a point-mass measure. We adopt the mathematical formalism used by Silva and Vinter [46], in which the controlled dynamic inclusion is the sum of a slow time velocity belonging to a set $F(x)$ and a fast time contribution coming from another set $G(x)d\mu$, where $d\mu$ is a vector valued measure.

$$\left\{ \begin{array}{rcl} dx & \in & F\big(x(t)\big)\,dt + G\big(x(t)\big)\,d\mu(dt) \\ x(0) & = & x_0. \end{array} \right. \tag{14}$$

There is a nontrivial and immediate issue as to whether the system (14) is well-defined. Bressan and Rampazzo [5, 6, 4] emphasized this point and introduced a solution concept based on the "graph completion" of the distribution function $u(t) = \mu([0, t])$. Independently, Murray [34] discovered this phenomenum by making "proper" extensions of Bolza-type functionals with Lagrangians that are not necessarily coercive (and therefore may have extended-valued Hamiltonians). This work is perhaps not as well-known as [5, 6, 4], but grew naturally from Rockafellar's earlier work in the context of full convexity [41], which will be mentioned again in §2.4. From the nonsmooth viewpoint that uses "infinite penalization" routinely, the dynamical system is a special case of the variational paradigm since it can be reformulated in those terms. Another goal of this proposal is to unite the two approaches under a common framework; this is not trivial since assumptions in one framework does not carry over well to the other.

The Bressan-Rampazzo solution concept is given in terms of the usual solution concept of a reparameterized problem in which the impulses or singularities of the measure μ are "blown-up" and so can be treated as if the "pause button" was held during time proportional to the total variation of the measure and the dynamics involving G could act. Although this method effectively resolves well-posedness and other issues involving solutions of (14), it is still natural to ask if a direct solution concept could be framed without recourse to the full time-reparameterization, and how "discrete-time" approximations to the solutions could be framed. These two issues have been addressed by my graduate student Stanislav Zabic and I [51], and it naturally opens the gateway for extending the standard theory [13] to impulsive systems. One of the difficulties of working with time reparameterizations is that sampling methods are buried in the new time, and is difficult to unearth when studying issues like invariance, equilibria, or asymptotic stability. A major goal of this proposal is to follow this trail where it clearly points; however, there are a lot technical details that still need to be resolved. Nonetheless, we have proven a sampling method [50, 51] that could open the way for recently developed nonsmooth methods to be applied here.

There appears to be many engineering applications in which a deeper understanding of impulsive systems would be helpful. For example, the study of hybrid systems is a very active area of current research [31], but these models often ignore the state-transition during jumps, which in many situations seems overly simplified. Another application area has recently been proposed by Artstein [1] related to singularly perturbations. I believe the completion and full achievement of the goals sketched above could lead to advances in these other areas (a similar viewpoint is expressed in [1]).

2.4 FCC systems

We introduced in §1.1 the Fully Convex Control (FCC) variational problem, which contains the Linear Quadratic Regulator (LQR) as a very special case. Of course LQR is the workhorse in applications of optimal control, but FCC extends well beyond LQR to incorporate "hard" constraints on control variables and general convex costs, situations where classical methods (typically involving the Ricatti equation) no longer apply. FCC deserves special treatment because it offers a broad class of problems that appear frequently in applications, and moreover, these problems can usually and actually be solved. By this, I refer to the hallmark property of convesity that local necessary conditions are also globally sufficient, and thus one "knows" when a solution is obtained since there are no local solutions that are not global. An underlying theme propounded by Rockafellar over many years (and espoused in Stephen Boyd's plenary talk at the last IEEE-CDC conference in Las Vegas) is that the practical, numerical, and theoretical difficulties in applications of a variational nature arise not between linear and nonlinear problems (as is the case in Dynamical Systems and PDE theory), but between convex and nonconvex problems. Convexity roughly plays the role in optimization that linearity enjoys in classical functional analysis. In fact, this is *precisely* true (in the appropriate sense) from the viewpoint of the Max-Plus algebra theory. However, it appears that this view is not prevalent or appreciated among most practical engineers, and there is still the need for theoretical work in the control context.

We sketched in §1.1 the results from [43, 44] that developed the basic FCC Hamilton-Jacobi theory, and here we propose extensions to (i) state-constrained problems, (ii) impulsive systems, and (iii) infinite horizon problems. These projects are presently taking shape in collaboration with Rafal Goebel, and in the case of (iii), also with Alain Rapaport.

(i) State Constraints are omnipresent in applications, but typically are ignored (when possible) or are handled by a variety of ad hoc methods. Even in the classical LQR case, state constraints pose challenging issues that classical methods cannot handle systematically. Engineers have expressed the need for a state constraint theory for LQR problems. Rockafellar developed optimality conditions (in the form of generalized Hamiltonian and Euler-

Lagrange equations) and a duality framework in the rather obscure paper [41]. Given the progress in nonsmooth analysis and Hamilton-Jacobi theory since then, the time is ripe to revisit this topic. The papers [43, 44] provide a clear road map for the type of results one can expect, but state constraints are always technically more challenging due largely to the lack of finiteness of the Hamiltonian. We mention there is significant work in Hamilton-Jacobi theory for systems with state constraints (by Soner, Vinter, Bardi, Clarke, Stern, and others), but the properties that hold on account of additional convexity assumptions has not yet been explored.

A related issue in this context is **(ii) impulsive systems**, since the dual arcs that appear in necessary conditions for problems with state constraints will typically be only of bounded variation; this observation was the motivation for Rockafellar's duality theory [41] in this context. As previously mentioned in §2.3, Murray [34] studied this approach and extended it to nonconvex systems, but as of yet, the Hamilton-Jacobi theory has not been developed here. We propose to develop an extension of LQR to allow impulsive and state-constraints in the problem formulation. Classical methods based on the Ricatti equation offer little guidance here, but it seems likely that the Hamilton-Jacobi approach will bear a full harvest of results. It might also be mentioned that the state interaction during the jump that was emphasized in §2.3 and which needs particular attention in that generality is not an issue under the FCC assumptions (which is yet another illustration of the special nature of FCC).

Finally, **(iii) infinite horizon** optimal control problems will be studied in the FCC framework. Rafal Goebel [28] has recently obtained some Hamilton-Jacobi related results in this context, and opens the way for additional research and extensions. We point out that infinite horizon problems are extremely important in economics and many engineering models, but is still rather mathematically undeveloped compared to the finite horizon case. For example, the excellent book by Carlson, Haurie, and Leizarowitz [8] contains little Hamilton-Jacobi theory and virtually no nonsmooth analysis. Furthermore, FCC problems have a rather prominent position in this book, but only under rather restrictive assumptions; there is no duality or utilization of value function techniques. Alain Rapaport has also worked considerably on infinite horizon problems in the context of renewable resources, and we are presently collaborating on developing and extending the turnpike properties worked out in [8] to multiple turnpikes in higher dimension. A nonsmooth approach based on value function analysis is very promising here.

2.5 Subgradient calculations and regularity properties of value functions

As mentioned in §1.2, we will continue to collaborate with Giovanni Colombo on finding explicit formulas for the subgradients of value functions in terms of the data. The motivation for this research stems from the enormous practical interest of finding feedback laws constructed from Lyapunov functions. It has become clear that nonsmooth analysis is a valuable tool for such constructions, but we shall not try to summarize here the large body of work in this area, except only to mention that the seminal paper [12] demonstrated the theoretical power of nonsmooth analysis to shape the direction of nonlinear control: discontinuous feedback laws are sometimes necessary. In this work and in subsequent innovative and substantial improvements by Rifford, Clarke, Ledyaev, Stern, and others, a key feature in the feedback construction makes use of the subgradients of the Lyapunov function. Since one of the main (theoretical) methods to construct Lyapunov functions is through a value function, it is useful to know more about the nature of these subgradients. The value functions being considered in this proposed research are relatively simple, but nonetheless the analysis is not so simple. Recall the value function (5) that has been studied in [16, 17], where explicit formulas have been derived. It is natural to continue our investigations into more complicated systems, and our subsequent investigations include new results for linear systems (that is, the dynamic equation is $\dot{x}(t) = Ax(t) + Bu(t)$) and where the target is convex. We need to finish and polish this result, and also perhaps include running costs, other side constraints, etc. These are related to a series of papers (e.g. [7]) by Cannarsa and his collaborators. Another closely related problem is to find the appropriate assumptions for which the minimal time function is prox-regular ([42]), a property well-understood for the distance function [39]. Semiconcavity now plays a fundamental role in stability theory (due to Rifford's result on the existence of semiconcave Lyapunov functions), and it is envisioned that prox-regularity will also, since it is a more general property that could be present in the absence of controllability.

3 Conclusion

3.1 Summary of activity

This proposal describes research problems in optimal control theory in five main categories: (1) One-sided Lipschitz multifunctions, (2) variational problems with time-delay, (3) impulsive systems, (4) fully convex control, and (5) subgradient formulas for value functions. Our methodology is based on nonsmooth and variational techniques that have been developed over the past three decades for general nonlinear control systems. Nonsmooth concepts and methods are now being used in control engineering design and is likely to expand much further. This proposal seeks to broaden and deepen the scope of these applications in other problem types under various non-standard assumptions.

A theory for One-Sided Lipschitz multifunctions, that if successfully established, would bring problems with dry friction under the purlieu of optimal control. Presently, to my knowledge, there is no maximum principle for such problems, although clearly many applied problems have such features. Variational problems with time-delays or with impulse behavior are two problem types that await the full onslaught of modern nonsmooth techniques in order to establish invariance characterizations, develop necessary conditions for optimality and a Hamilton-Jacobi theory, construct feedback maps, etc. The fully convex control problem is a generalization of the Linear Quadratic Regulator but allows for realistic constraints. There are natural mathematical models in this framework that involve state constraints, impulsive behavior, and infinite horizons, and these all pose mathematical challenges that are engaged in this proposal. Finally, there is the problem of calculating subgradients directly in terms of the data for new classes of value functions, an issue that has surfaced perhaps only because nonsmooth analysis has developed so completely. Such calculations, however, could be quite valuable in constructing feedback laws and for providing insight for the design of velocity and target sets.

3.2 Broader impact

Direct applications of control theory are omnipresent in our daily lives, and control engineering is having a momentous impact in the development of modern technology. The simple phrase “things move, let’s move them better” could be the motto for enhancing economic development through increasing efficiency. Of interest, then, is (1) to understand how things move, and (2) the dependence on parameters to improve performance. The former relies upon the laws of nature, but can be circumscribed in the modeling phase by the existing mathematical theory (as it is with dry friction). The latter is optimization, an area that classical differentiability concepts are not well-suited (the min or max of differentiable functions will not in general be differentiable). Nonsmooth and variational analysis has largely been developed to fill this void, and it is not surprising that control engineers are increasingly discovering and using these powerful mathematical tools.

Nonetheless, despite its high mathematical content and challenges, mathematical control theory is not a subject well-represented in most US mathematical departments (LSU is somewhat of an exception). Its influence on the general mathematical curriculum is thus not particularly strong, although it continues to challenge existing mathematical theories and has a high overall mathematical content. The LSU Mathematics Department was recently awarded additional university support to build a program of excellence. This means the department's research faculty and graduate student support will expand by roughly one third over the next four years (from 44 to 60 research faculty and from 60 to 100 graduate students). The PI has for several years been involved in curriculum reform, and has developed interdisciplinary math graduate courses that are taken by both math and graduate students from other departments. He envisions going much further by requesting that some of the new resources be allocated to creating a "Mathematical Control Center" at LSU, in which an intensive mathematical program will be devised with interdisciplinary components to train and to provide additional mathematical support for other control theorists on campus. The new university support was obtained at least in part by PI's activity in applied math areas, and the department is generally supportive in these efforts. The funding of the present proposal will provide resources to keep the PI's research program active and broaden his exposure to engineering applications that will be incorporated into the larger educational platform at LSU.

As mentioned in the narrative, the PI has three advanced graduate students who have already co-authored papers and are expected to graduate in the spring 2006. A fourth unfortunately ran into visa problems and will be delayed. The PI has a recent record on attracting good graduate students, and will teach a graduate course on the calculus of variations and optimal control in the fall 2004 semester to recruit new ones into this exciting field.

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Biographical sketch

(i) Professional preparation

University of Pittsburgh, Pittsburgh, PA	Mathematics	BS, 1978
University of Washington, Seattle, WA	Mathematics	Ph.D. 1988
Imperial College of Science and Technology, London, England	Mathematics	July 1988-Jan. 1989
International Institute of Applied Systems Analysis, Laxenburg, Austria	Mathematics	Feb. 1989-Aug. 1989
Centre de Recherches Mathématiques, Université de Montréal, Montréal, Canada	Mathematics	Sept 1989-Aug. 1990
Centre de Recherches Mathématiques, Université de Montréal, Montréal, Canada	Mathematics	Summers 0f 1991-4

(ii) Appointments

Professor	Department of Mathematics Louisiana State University	2001-present
Associate Professor	Department of Mathematics Louisiana State University	1995-2001
Assistant Professor	Department of Mathematics Louisiana State University	1990-1995

(iiia) Publications, most relevant to the current proposal

(1) "The decoupling technique for continuously varying time delay systems" (with N.L. Ortiz), Set Valued Analysis, to appear, 2004.
(2) "Convexity in Hamilton-Jacobi theory I: dynamics and duality" (with R.T. Rockafellar), SIAM Journal of Control and Optimization, **39**, 1323-1350, 2001.
(3) "Convexity in Hamilton-Jacobi theory II: envelope representations" (with R.T. Rockafellar), SIAM Journal of Control and Optimization, **39**, 1351-1372, 2001.
(4) *Nonsmooth Analysis and Control theory* (with F. H. Clarke, Yu. Ledyaev, and R.J. Stern), Graduate Texts in Mathematics v. **178**, Springer-Verlag, New York, 1998.
(5) "Proximal analysis and the minimal time function" (with Yu Zhuang), SIAM Journal of Control and Optimization}, **36**, 1048-1072, 1998.

(iii.b) Publications, other significant publications

(6) "Necessary conditions for functional differential inclusions" (with F.H. Clarke), Applied Mathematics and Optimization, **34**, 51-78, 1996.
(7) "Qualitative properties of trajectories of control systems: a survey (with F. H. Clarke, Yu. Ledyaev, and R.J. Stern), Journal of Dynamical and Control Systems, **1**, 1-48, 1995.

(8) “Proximal smoothness and the lower C2 property” (with F.H. Clarke and R.J. Stern), Journal of Convex Analysis, **2**, 117-144, 1995.
(9) “Proximal analysis and minimization principles” (with F. H. Clarke and Yu. Ledyaev), Journal of Math. Analysis and Applications, **196**, 722—735, 1995.
(10) “The exponential formula for the reachable set of a Lipschitz differential inclusion}, SIAM J. Control Optimization, **28,** 1148—1161, 1988.

(iv) Synergistic activities

(1) Founder of the LSU Mathematics Consultation Clinic (MCC). The MCC is a program begun five years ago at LSU that exposes students to real-world applications of mathematics. It is centered on Math 4020, the Capstone course, and each semester undergraduate teams tackle a project proposed by a local industry, government, or educational research group.
(2) Director of the Math Summer Tune-Up Camp workshop. We have run the summer camp the last two years funded by a Louisiana Board of Regents grant. It runs between summer and fall terms for four weeks in August, and is targeted at beginning graduate students in other disciplines. Morning sessions are devoted to faculty lectures, and the afternoon problem sessions are supervised by experienced math graduates students. Last year, 50 students representing 10 departments across campus applied to the camp, and 30 completed the program.
(3) Curriculum Reform. I have initiated several reforms of the graduate program at LSU over the past three years, including a restructuring of the core graduate courses. Developed the graduate Ordinary Differential Equations for this purpose, and also an undergraduate course in Discrete Dynamical Systems (Chaos and Fractals).

(v.a) Collaborators, co-authors

F.H. Clarke (Institut Desargues, Univ Lyon I, France)
G. Colombo (Univ. di Padova, Italy)
T. Donchev (University of Architecture& Civil Engineering, Sofia, Bulgaria)
E. Farkhi (Tel Aviv University, Israel)
Yu. Ledyaev (Western Michigan University, Kalamazoo)
P.D. Loewen (Univ. of British Columbia, Canada)
R.T. Rockafellar (Univ. of Washington, Seattle)
R.J. Stern (Concordia Univ., Montreal, Canada)

(v.b) Graduate and Post-doctorate advisers

Craig Knuckles, Ph.D. 1995; Vinicio Rios, Ph,D, (expected), 2005; Stanislav Zabic, (expected), 2005; Norma Ortiz, (expected), 2005; Khalid al-Jammari, (expected), 2005.

(v.c) Thesis and Post-graduate sponsor
R.T. Rockafellar (Ph.D. adviser), R.B. Vinter (post-graduate adviser), F.H. Clarke (post-graduate adviser)

SUMMARY PROPOSAL BUDGET

YEAR 1

ORGANIZATION **Louisiana State University & Agricultural and Mechanical College**				FOR NSF USE ONLY PROPOSAL NO.	DURATION (months) Proposed	Granted
PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR **Peter R Wolenski**				AWARD NO.		
A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-months CAL	ACAD	SUMR	Funds Requested By proposer	Funds granted by NSF (if different)	
1. Peter R Wolenski - Professor	0.00	0.00	2.00	$ 15,351	$	
2.						
3.						
4.						
5.						
6. (0) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	0		
7. (1) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	0.00	2.00	15,351		
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)						
1. (0) POST DOCTORAL ASSOCIATES	0.00	0.00	0.00	0		
2. (0) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	0		
3. (1) GRADUATE STUDENTS				15,000		
4. (0) UNDERGRADUATE STUDENTS				0		
5. (0) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				0		
6. (0) OTHER				0		
TOTAL SALARIES AND WAGES (A + B)				30,351		
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				3,454		
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				33,805		
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)						
TOTAL EQUIPMENT				0		
E. TRAVEL 1. DOMESTIC (INCL. CANADA, MEXICO AND U.S. POSSESSIONS)				1,500		
2. FOREIGN				0		
F. PARTICIPANT SUPPORT COSTS 1. STIPENDS $ 0 2. TRAVEL 0 3. SUBSISTENCE 0 4. OTHER 0						
TOTAL NUMBER OF PARTICIPANTS (0) TOTAL PARTICIPANT COSTS				0		
G. OTHER DIRECT COSTS						
1. MATERIALS AND SUPPLIES				1,000		
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				0		
3. CONSULTANT SERVICES				2,000		
4. COMPUTER SERVICES				0		
5. SUBAWARDS				0		
6. OTHER				0		
TOTAL OTHER DIRECT COSTS				3,000		
H. TOTAL DIRECT COSTS (A THROUGH G)				38,305		
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE) MTDC (Rate: 47.0000, Base: 38305)						
TOTAL INDIRECT COSTS (F&A)				18,003		
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				56,308		
K. RESIDUAL FUNDS (IF FOR FURTHER SUPPORT OF CURRENT PROJECTS SEE GPG II.C.6.j.)				0		
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				$ 56,308	$	
M. COST SHARING PROPOSED LEVEL $ Not Shown	AGREED LEVEL IF DIFFERENT $					
PI/PD NAME **Peter R Wolenski**				FOR NSF USE ONLY INDIRECT COST RATE VERIFICATION		
ORG. REP. NAME* **James bates**				Date Checked	Date Of Rate Sheet	Initials - ORG

 *ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET

SUMMARY PROPOSAL BUDGET YEAR 2

ORGANIZATION: Louisiana State University & Agricultural and Mechanical College				FOR NSF USE ONLY	
PROPOSAL NO.				DURATION (months) Proposed	Granted
PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR: Peter R Wolenski				AWARD NO.	

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-months CAL	ACAD	SUMR	Funds Requested By proposer	Funds granted by NSF (if different)
1. Peter R Wolenski - none	0.00	0.00	2.00	$ 15,965	$
2.					
3.					
4.					
5.					
6. (0) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	0	
7. (1) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	0.00	2.00	15,965	
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)					
1. (0) POST DOCTORAL ASSOCIATES	0.00	0.00	0.00	0	
2. (0) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	0	
3. (1) GRADUATE STUDENTS				15,000	
4. (0) UNDERGRADUATE STUDENTS				0	
5. (0) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				0	
6. (0) OTHER				0	
TOTAL SALARIES AND WAGES (A + B)				30,965	
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				3,592	
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				34,557	
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)					
TOTAL EQUIPMENT				0	
E. TRAVEL 1. DOMESTIC (INCL. CANADA, MEXICO AND U.S. POSSESSIONS)				1,500	
2. FOREIGN				2,000	
F. PARTICIPANT SUPPORT COSTS					
1. STIPENDS $ 0					
2. TRAVEL 0					
3. SUBSISTENCE 0					
4. OTHER 0					
TOTAL NUMBER OF PARTICIPANTS (0) TOTAL PARTICIPANT COSTS				0	
G. OTHER DIRECT COSTS					
1. MATERIALS AND SUPPLIES				3,000	
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				0	
3. CONSULTANT SERVICES				2,000	
4. COMPUTER SERVICES				0	
5. SUBAWARDS				0	
6. OTHER				0	
TOTAL OTHER DIRECT COSTS				5,000	
H. TOTAL DIRECT COSTS (A THROUGH G)				43,057	
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE) MTDC (Rate: 47.0000, Base: 43057)					
TOTAL INDIRECT COSTS (F&A)				20,237	
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				63,294	
K. RESIDUAL FUNDS (IF FOR FURTHER SUPPORT OF CURRENT PROJECTS SEE GPG II.C.6.j.)				0	
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				$ 63,294	$
M. COST SHARING PROPOSED LEVEL $ Not Shown	AGREED LEVEL IF DIFFERENT $				

PI/PD NAME: Peter R Wolenski	FOR NSF USE ONLY		
	INDIRECT COST RATE VERIFICATION		
ORG. REP. NAME*: James bates	Date Checked	Date Of Rate Sheet	Initials - ORG

 *ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET

SUMMARY PROPOSAL BUDGET YEAR 3

ORGANIZATION **Louisiana State University & Agricultural and Mechanical College**			FOR NSF USE ONLY PROPOSAL NO.	DURATION (months) Proposed	Granted
PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR **Peter R Wolenski**			AWARD NO.		

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-months CAL	ACAD	SUMR	Funds Requested By proposer	Funds granted by NSF (if different)
1. Peter R Wolenski - none	0.00	0.00	2.00	$ 16,604	$
2.					
3.					
4.					
5.					
6. (0) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	0	
7. (1) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	0.00	2.00	16,604	
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)					
1. (0) POST DOCTORAL ASSOCIATES	0.00	0.00	0.00	0	
2. (0) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	0	
3. (1) GRADUATE STUDENTS				15,000	
4. (0) UNDERGRADUATE STUDENTS				0	
5. (0) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				0	
6. (0) OTHER				0	
TOTAL SALARIES AND WAGES (A + B)				31,604	
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				3,736	
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				35,340	
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)					
TOTAL EQUIPMENT				0	
E. TRAVEL 1. DOMESTIC (INCL. CANADA, MEXICO AND U.S. POSSESSIONS)				1,500	
2. FOREIGN				2,000	
F. PARTICIPANT SUPPORT COSTS 1. STIPENDS $ 0 2. TRAVEL 0 3. SUBSISTENCE 0 4. OTHER 0					
TOTAL NUMBER OF PARTICIPANTS (0) TOTAL PARTICIPANT COSTS				0	
G. OTHER DIRECT COSTS					
1. MATERIALS AND SUPPLIES				3,000	
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				0	
3. CONSULTANT SERVICES				2,000	
4. COMPUTER SERVICES				0	
5. SUBAWARDS				0	
6. OTHER				0	
TOTAL OTHER DIRECT COSTS				5,000	
H. TOTAL DIRECT COSTS (A THROUGH G)				43,840	
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE) MTDC (Rate: 47.0000, Base: 43840)					
TOTAL INDIRECT COSTS (F&A)				20,605	
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				64,445	
K. RESIDUAL FUNDS (IF FOR FURTHER SUPPORT OF CURRENT PROJECTS SEE GPG II.C.6.j.)				0	
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				$ 64,445	$

M. COST SHARING PROPOSED LEVEL $ **Not Shown** AGREED LEVEL IF DIFFERENT $

PI/PD NAME **Peter R Wolenski**	FOR NSF USE ONLY INDIRECT COST RATE VERIFICATION		
ORG. REP. NAME* **James bates**	Date Checked	Date Of Rate Sheet	Initials - ORG

 ***ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET**

SUMMARY PROPOSAL BUDGET — Cumulative

ORGANIZATION: Louisiana State University & Agricultural and Mechanical College				FOR NSF USE ONLY	
				PROPOSAL NO.	DURATION (months): Proposed / Granted
PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR: Peter R Wolenski				AWARD NO.	

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-months CAL	ACAD	SUMR	Funds Requested By proposer	Funds granted by NSF (if different)
1. Peter R Wolenski - none	0.00	0.00	6.00	$ 47,920	$
2.					
3.					
4.					
5.					
6. ( ) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	0	
7. (1) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	0.00	6.00	47,920	
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)					
1. (0) POST DOCTORAL ASSOCIATES	0.00	0.00	0.00	0	
2. (0) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	0	
3. (3) GRADUATE STUDENTS				45,000	
4. (0) UNDERGRADUATE STUDENTS				0	
5. (0) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				0	
6. (0) OTHER				0	
TOTAL SALARIES AND WAGES (A + B)				92,920	
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				10,782	
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				103,702	
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)					
TOTAL EQUIPMENT				0	
E. TRAVEL 1. DOMESTIC (INCL. CANADA, MEXICO AND U.S. POSSESSIONS)				4,500	
2. FOREIGN				4,000	
F. PARTICIPANT SUPPORT COSTS					
1. STIPENDS $ 0					
2. TRAVEL 0					
3. SUBSISTENCE 0					
4. OTHER 0					
TOTAL NUMBER OF PARTICIPANTS (0) TOTAL PARTICIPANT COSTS				0	
G. OTHER DIRECT COSTS					
1. MATERIALS AND SUPPLIES				7,000	
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				0	
3. CONSULTANT SERVICES				6,000	
4. COMPUTER SERVICES				0	
5. SUBAWARDS				0	
6. OTHER				0	
TOTAL OTHER DIRECT COSTS				13,000	
H. TOTAL DIRECT COSTS (A THROUGH G)				125,202	
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE)					
TOTAL INDIRECT COSTS (F&A)				58,845	
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				184,047	
K. RESIDUAL FUNDS (IF FOR FURTHER SUPPORT OF CURRENT PROJECTS SEE GPG II.C.6.j.)				0	
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				$ 184,047	$
M. COST SHARING PROPOSED LEVEL $ Not Shown	AGREED LEVEL IF DIFFERENT $				

PI/PD NAME: Peter R Wolenski	FOR NSF USE ONLY		
	INDIRECT COST RATE VERIFICATION		
ORG. REP. NAME*: James bates	Date Checked	Date Of Rate Sheet	Initials - ORG

C *ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET

Budget Justification Page

A. Senior Personnel

1. The PI requests two months summer salary for each year to work on this project. Base salary is $69,080; anticipated salary increases of 4% are included for Years 2 and 3.

B. Other Personnel

3. The PI requests $15,000 each year to support one graduate student to assist with the project. There is currently three Ph.D. students that will complete their degree requireents in May, 2005, and there are new students interested working with the PI. He will choose the most qualified student for the support.

C. Fringe benefits are included at 22.5% of PI salary request.

E. Travel

1. Domestic Travel: The PI requests $1500 in Years 1-3 to attend conferences. Typically, he attends the annual IEEE-CDC held in the second week of December, and plans to organize sessions and present his research there. He also plans to attend other control theory conferences.

2. Foreign Travel: The PI requests $2000 in each of Years 2 and 3 to support travel to conferences and visits to collaborators in France (Clarke, Rapaport), Italy (Colombo), and Bulgaria (Donchev)

G. Other Direct Costs

1. The PI requests $1000 in year 1 and $3000 in years 2 and 3 to purchase books, renew Matlab and other software licenses, and other project related supplies.

3. The PI requests $2000 each year to support visitors to collaborate on this research. Visitors will likely include R.J. Stern, F.H. Clarke, A. Rapaport, G. Colombo, T. Donchev, and R. Goebel.

I. Indirect Costs are calculated at LSU's federally negotiated rate of 47% of MTDC.

Current and Pending Support

(See GPG Section II.D.8 for guidance on information to include on this form.)

The following information should be provided for each investigator and other senior personnel. Failure to provide this information may delay consideration of this proposal.

Investigator: **Peter Wolenski**

Other agencies (including NSF) to which this proposal has been/will be submitted.

Support: ☒ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support

Project/Proposal Title: **Interdisciplinary education, outreach, and research in Control Theory at LSU**

Source of Support: **Louisiana State Board of Regents**

Total Award Amount: $ **103,000** Total Award Period Covered: **06/01/02 - 06/30/04**

Location of Project: **Louisiana State University**

Person-Months Per Year Committed to the Project. Cal: **0.00** Acad: **0.00** Sumr: **0.00**

Support: ☒ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support

Project/Proposal Title: **Enhancement of interdisciplinary, industrial, and applied mathematics education and outreach at LSU**

Source of Support: **Louisiana Board of Regents**

Total Award Amount: $ **214,271** Total Award Period Covered: **06/01/02 - 06/30/04**

Location of Project: **Louisiana State University**

Person-Months Per Year Committed to the Project. Cal: **0.00** Acad: **0.00** Sumr: **0.00**

Support: ☐ Current ☒ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support

Project/Proposal Title: **Nonsmooth mathods in optimal control theory**

Source of Support: **NSF**

Total Award Amount: $ **184,047** Total Award Period Covered: **06/01/04 - 07/31/07**

Location of Project: **Louisiana State University**

Person-Months Per Year Committed to the Project. Cal: **0.00** Acad: **0.00** Sumr: **2.00**

Support: ☐ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support

Project/Proposal Title:

Source of Support:

Total Award Amount: $ Total Award Period Covered:

Location of Project:

Person-Months Per Year Committed to the Project. Cal: Acad: Sumr:

Support: ☐ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support

Project/Proposal Title:

Source of Support:

Total Award Amount: $ Total Award Period Covered:

Location of Project:

Person-Months Per Year Committed to the Project. Cal: Acad: Summ:

*If this project has previously been funded by another agency, please list and furnish information for immediately preceding funding period.

 USE ADDITIONAL SHEETS AS NECESSARY

FACILITIES, EQUIPMENT & OTHER RESOURCES

FACILITIES: Identify the facilities to be used at each performance site listed and, as appropriate, indicate their capacities, pertinent capabilities, relative proximity, and extent of availability to the project. Use "Other" to describe the facilities at any other performance sites listed and at sites for field studies. USE additional pages as necessary.

Laboratory:

Clinical:

Animal:

Computer:

Office:

Other:

MAJOR EQUIPMENT: List the most important items available for this project and, as appropriate identifying the location and pertinent capabilities of each.

OTHER RESOURCES: Provide any information describing the other resources available for the project. Identify support services such as consultant, secretarial, machine shop, and electronics shop, and the extent to which they will be available for the project. Include an explanation of any consortium/contractual arrangements with other organizations.

The PI and his graduate students are well-equipped with computer hrdware and software, thanks to recent Louisiana Board of Regents grants obtained by the PI.