MATHEMATICS 300 — SPRING 2006

Introduction to Mathematical Reasoning

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INSTRUCTOR'S NOTES

(April 12, 2006)

1 Homework assignments No. 8, 9, 10, 11 (due dates: Wed. Mar. 29, Wed. April 5, Wed. April 12, Wed. April 19)

1.1 Homework assignment No. 8, due on Wed., March 29

Assignment No. 7 was pretty long. You probably did not do all the problems, so here in Assignment No. 8 I am giving you a second chance to do them. And Assignment No. 8 is quite short.

1. If you didn't hand in all the problems of Homework No. 7, do as many as you can of the ones you still owe.
2. Book, page 109, Problem 15, Parts (c) and (f).
3. Book, pages 116-117, Problems 5(b)(c) and Problem 15(a).

1.2 Homework assignment No. 9, due on Wed., April 5

1. Let a, b, c be nonzero integers. Suppose that a and b are coprime, b and c are coprime, an d a and c are coprime. Prove that a and bc are coprime. (Recall that two integers α, β are said to be ***coprime*** if the gratest common divisor of α and β is equal to 1.)
2. If a, b, c are nonzero integers, we say that c is the ***least common multiple of*** a ***and*** b if (i) $c > 0$, (ii) a divides c, (iii) b divides c, and (iv) if x is any natural number such that a divides x, and b divides x, it follows that c divides x. In symbolic language, using "LCM" as an abbreviation for "least common multiple", and writing "$c = LCM(a,b)$" for "c is the LCM of a and b":

$$(\forall a,b,c\in\mathbb{Z})\Big(c=LCM(a,b)\Leftrightarrow((c>0\wedge(a|c\wedge b|c))\wedge(\forall x\in\mathbb{N})((a|x\wedge b|x)\Rightarrow c\leq x))\Big)$$

(NOTE: "$(\forall a,b,c\in\mathbb{Z})$" is an abbreviation of "$(\forall a\in\mathbb{Z})(\forall b\in\mathbb{Z})(\forall c\in\mathbb{Z})$".)

Let a, b be natural numbers. Let d be the greatest common divisor of a and b, and let m be the least common multiple of a and b. Prove that $md = ab$. (Note: this is problem 11(i) in the book, pages 65-66. You may wish to look at the previous questions in that problem, and use them as hints for your solution. Also, you may find it useful to compute the GCD and the LCD of a and b in a few examples—say, $a = 12$ and $b = 15$, $a = 36$ and $b = 61$, $a = 30$ and $b = 40$—to have a better idea of what is going on before you tackle the general case.)

3. Book, pages 76, 77, 78, problems 1(b)(d)(e)(f), 3(nostarred items), 4(nostarred items), 6(b)(d)(e), 7(b)(d)(e)(f), 8, 9(b)(d)(f), 19(non-starred items).

1.3 Homework assignment No. 10, due on Wed., April 12

1. Book, page 77, problem 18.
2. Book, pages 84-85: problems 11(d)(e), 13(b)(c), 15(b), 17(c)(f)(g)(h).

1.4 Homework assignment No. 11, due on Wed., April 19

1. Book, page 142-3-4-5, problems 2, 10 (non-starred items), 15, 18, 20 (c)(e)(f)(g).
2. Book, pages 150-1-2-3-4: problems 9, 13, and 16(a)(b)(d)(e).