MATHEMATICS 300 — FALL 1998

Introduction to Mathematical Reasoning

H. J. Sussmann

HANDOUT NO. 7

September 30, 1998

A REMINDER

The **first midterm exam** is going to be on **Wednesday, October 14**.

READING FOR OCTOBER 5-7

Book, Sections 2.1 and 2.2.

HOMEWORK ASSIGNMENT NO. 4
DUE ON WEDNESDAY, OCTOBER 7

a. Problems 1(c), 3, 9 and 10 (nonstarred parts only), in pages 54-55-56-57.

b. Using the definitions of "continuous function" and "limit," translate the sentence
(1) the function f given by $f(x) = x^4 + x^3 - 2x$ is continuous on $\mathbf{R}$
into a formula of the first-order predicate calculus language of real arithmetic (in which the universe of discourse is $\mathbf{R}$, the set of real real numbers) that only involves the following symbols:

> $($, $)$, a , x , ε , δ , $\forall$, $\exists$, $=$, $>$, $<$, $\wedge$, $\vee$, $\sim$, $\Longrightarrow$, 0 , 1 , 2 , 3 , 4 , $|$, $+$, $-$, and superscripts such as ^{2} , ^{3} , ^{4} to indicate powers

AND NOTHING ELSE, and **prove** (1).

c. In this problem, "number" means "natural number," so the numbers are 1, 2, 3, etc., and "square" means "square of a natural number." Prove or disprove each of the following statements:
(1) Every number is the difference of two squares.
(2) Every even number is the difference of two squares.
(3) Every odd number is the difference of two squares.

d. In this problem, "number" means "nonnegative integer," so the numbers are 0, 1, 2, 3, etc., and "square" means "square of a nonnegative integer." Prove or disprove each of the following statements:
(1) Every number is the difference of two squares.
(2) Every even number is the difference of two squares.
(3) Every odd number is the difference of two squares.

A CORRECTION

In Review Question No. 87 (Handout No. 6), we really need a language where, in addition to the predicate "father," there is also a symbol for the predicate "equal." So the question should read as follows:

87. In this problem we work with a first-order predicate calculus with two two-variable symbols, namely, p and $=$. The universe of discourse is the set of all people. We interpret $p(x, y)$ to mean "x is y's father." For the other predicate symbol, we write "$x = y$" rather than "$= (x, y)$," and interpret $x = y$ to mean that x and y are the same person. (So the symbols are $\forall$, $\exists$, (,), $\wedge$, $\vee$, $\sim$, $\Longrightarrow$, p, $=$, the comma, the variables $x, y, \ldots$, **and nothing else**. In particular, the symbol ! is **not** part of this language.)
Translate each of the following sentences into symbolic language:
    a. Everybody has a father.
    b. Everybody has one and only one father.
    c. Nobody is his own father.
    d. The father of the father of a person is not the father of that person.
    e. Some people are fathers and some people are not.

MORE REVIEW QUESTIONS

88. Explain what the word "subset" means.
89. What is the empty set?
90. Is it true or false that $(\exists X)(X \text{ is a set} \wedge (\forall Y)(Y \text{ is a set} \Longrightarrow Y \subseteq X))$?
91. Is it true or false that $(\exists X)(X \text{ is a set} \wedge (\forall Y)(Y \text{ is a set} \Longrightarrow Y \cup X = X))$?
92. Is it true or false that $(\exists X)(X \text{ is a set} \wedge (\forall Y)(Y \text{ is a set} \Longrightarrow Y \cap X = X))$?
93. Is it true or false that $(\exists X)(X \text{ is a set} \vee (\forall Y)(Y \text{ is a set} \Longrightarrow (\sim Y \subseteq Y)))$?
94. Is it true or false that $(\exists X)(X \text{ is a set} \wedge (\forall Y)(Y \text{ is a set} \Longrightarrow X \subseteq X))$?
95. Is it true or false that $(\exists X)(X \text{ is a set} \wedge (\forall Y)(Y \text{ is a set} \Longrightarrow Y \subseteq Y))$?
96. Is it true or false that $(\exists X)(X \text{ is a set} \wedge (\forall Y)(Y \text{ is a set} \wedge Y \subseteq X))$?
97. Is it true or false that $(\exists X)(X \text{ is a set} \wedge (\forall Y)(Y \text{ is a set} \vee Y \subseteq X))$?
98. Explain what the "power set" of a set is.
99. Suppose n is a nonnegative integer and S is a set with n elements. How many elements does the power set of S have?
100. Define "union" of sets.
101. Define "intersection" of sets.
102. Define "difference" of sets.
103. Define "complement."
104. Union, intersection, difference and complement are set operations. How many arguments does each of those operations take? (That is, which ones are unary, which ones are binary?)
105. Is it true or false that if S is a set then $\emptyset \subseteq S$?
106. Is it true or false that if S is a set then $\emptyset \in S$?
107. What is a "paradox"?
108. Explain what the *Russell paradox* is, and why it is called a "paradox."
109. Suppose 70% of all people like coffee and 60% like tea. What can you say about the percentage of people who like both coffee and tea?