MATHEMATICS 300 — SPRING 1997

Introduction to Mathematical Reasoning

Instructor: H. J. Sussmann

HANDOUT NO. 7. February 20, 1997

The **INSTRUCTOR'S OFFICE HOURS ARE:** Monday and Thursday, 2:50 to 4:10 p.m., in Hill 704.

The **FIRST MIDTERM EXAM** will be on **Thursday, February 27.**

READING ASSIGNMENTS SO FAR

- For the week of January 28 and 30, you have to read *all of Chapter 1.* Plus, of course, the first handout.
- For the week of February 4 and 6, you have to read *Sections 1, 2 and 3 of Chapter 2.*
- For the week of February 11 and 13, you have to read *Sections 2.4, 3.1 and 3.2.*
- For the week of February 18 and 20, you have to read *Sections 3.3 and 3.4.*
- No new reading assignment for the week of February 25 and 27.

HOMEWORK ASSIGNMENTS SO FAR

(1) *Originally due on Tuesday, January 28. The deadline was subsequently extended to February 4.* Page 14, Problem 4; Page 15, Problem 7; Page 26, Problem 10; Page 27, Problem 11; Page 37, Problems 2 and 8.

(2) *Due Tuesday, February 4.* Page 14, Problem 5; Page 47, Problems 1 (parts (a), (j), (q)), 2, and 3; Page 48, Problems 5 (parts (a), (c)) and 7; Page 49, Problems 11 and 12.

(3) *Due Tuesday, February 11.* Page 56, Problem 1, parts (a), (b), (f), (g), (h), (j), (l). Problem 4, parts (a), (b), (c), (e), (h), (i), (j). Page 60-61, Problem 2. Page 61, Problems 3,4. And, in addition, the following:

(A) Let S be the set of all words consisting of exactly three vowels and three consonants, with repetition allowed for the vowels but not for the consonants. (For example: *arabap*, *benito*, *yaaexz* and *xaeiyz* belong to S, but *bavenii*, *baeiou*, *babeic* and *gugugu* do not. Recall that in the English alphabet there are 5 vowels and 21 consonants.)

(i) Determine how many members S has.

(ii) Determine what percentage of the words in S contain the letter "a". Is it higher or lower than 50%?

(4) *Due Tuesday, February 18.* Page 70, problems 4, 6, 8, 9. Page 72, problem 14. Page 73, problem 16. Page 76, problems 6 and 9.

(*) **Optional homework assignment**, *due Tuesday, February 25.* Write a short essay (about one page) discussing the February 7 New York Times article of Handout No. 4. In particular, try to address the following two issues: (a) would the authors of the S.A.T. question have made the mistake they made if they had followed the rules on writing mathematics given in Handout No. 1 of our course? (b) can you find *another mistake* in the question, different from the one found by Mr. Rizzio, which gives a different reason for (D) to be the right answer, even if it had been stipulated that $a > 0$?

(5) *Due Tuesday, February 25.* (Optional.) Discuss the following definitions of "interval." In each case, decide first of all if the proposed definition is meaningful or meaningless. If it is meaningful, determine whether what it means is the right thing or something different. To show that the purported definition does not say what it ought to say, you should give an example of a set that is an interval according to the correct definition but not according to the proposed definition, or of a set that is an interval according to the proposed definition but not according to the correct one. Discuss how this definition would be expressed in formal language; assign it a grade on a scale from 0 to 10, and justify your grade. Do not hesitate to be very critical as you have to be, but do not make criticisms unless you can justify them.
We will do some of these in class. You should do the others.

1. If there exists a set X of real numbers, if $a \in X$ and $b \in X$, and $a < c < b$. Therefore $c \in X$ within the interval of a and b.
2. Let I be a set. Let $a, b \in \mathbf{R}$ such that $a < b$. Let $a, b \in I$. (a, b) is an *interval* if and only if there exists $c \in \mathbf{R}$ such that $a < c < b$ and $c \in I$.
3. An interval is a set of real numbers I such that, whenever $x \in I$, $y \in I$, $z \in \mathbf{R}$, and $x < z < y$, it follows that $z \in I$.
4. An *interval* is a set of real numbers I such that, whenever $x \in I$, $y \in I$, $z \in I$, and $x < z < y$, it follows that $z \in I$.
5. Interval is a set of numbers such that if we pick two numbers of this set then every number between those two numbers is also a member of the set.
6. An *interval* is a set I of real numbers such that, whenever $x \in I$, $y \in I$, $z \in \mathbf{R}$, and $x < y < z$, it follows that $z \in I$.
7. An *interval* is a set I of real numbers having the property that, if you pick x and y in I and z in $\mathbf{R}$ such that $x < z < y$, then it follows that z belongs to I.
8. An *interval* is a set I of real numbers having the property that, whenever a real number lies between two members of I, it follows that this number is also in I.
9. An *interval* is a set of real numbers having the property that, whenever a real number lies between two members of I, it follows that this number is also I.
10. An *interval* is a set I of real numbers such that, if x and y are in I, then there exists a z in I such that $x < z < y$.
11. An *interval* I is a set of real numbers such that, if $a, b \in I$, $c \in \mathbf{R}$, and $a < c < b$, then $c \in I$.
12. Interval. Allowable range of numbers that are real. Interval implies that to be valid our variable must fall between the intervals endpoints. Depending on syntax the endpoints may be included $0 \leq x \leq 1$. Between 0 & 1 w/being valid.