MATHEMATICS 300 — FALL 1999

Introduction to Mathematical Reasoning

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HANDOUT NO. 6 *(October 11, 1999)*

READING FOR THE FIRST TWELVE DAYS OF CLASS

(that is, for Sept. 2-9-13-16-20-23-27-30 and October 4-7-11-14)

- the book's "Preface for the Student,"
- the book's Chapters 1 and 2;
- the instructor's Handouts Nos. 1, 2, 3, 4, 5 and 6.

THE FIRST MIDTERM TEST

The test is going to be on

Thursday, October 21.

HOMEWORK ASSIGNMENT NO. 5

This assignment is due on Monday October 18.

- Book, problems 2.8, 2.11, 2.19, 2.22, 2.23 and 2.25.

REVIEW QUESTIONS

1. State the quadratic formula and show how to derive it.
2. State the triangle inequality and show how to derive it.
3. State the arithmetic-geometric mean inequality and show how to derive it.
4. Explain what is wrong with the following statement: if x and y are real numbers then $\sqrt{xy} \leq \frac{x+y}{2}$.
5. Explain what is wrong with the following "proof" that $\sqrt{xy} \leq \frac{x+y}{2}$ whenever $x \geq 0$ and $y \geq 0$:

$\sqrt{xy} \leq \frac{x+y}{2}$,

Now, square both sides:

$xy \leq \frac{(x+y)^2}{4}$,

$4xy \leq (x+y)^2$,

$4xy \leq x^2 + 2xy + y^2$,

$0 \leq x^2 - 2xy + y^2$,

$0 \leq (x-y)^2$,

and this is true because the square of every real number is ≥ 0. $\diamondsuit$

6. Explain the difference between a “set” and a “list.”
7. Define “union.”
8. Define “intersection.”
9. Define “subset.”
10. Define what it means for two sets to be “disjoint.”
11. Define “Cartesian product.”
12. Explain what a “function” is.
13. Explain what the “graph” of a function is.
14. Explain what an “increasing function” is.
15. Explain what it means for a set to be “bounded.”
16. Explain what it means for a function to be “bounded.”
17. What is a “field”?
18. What is an “ordered field”?
19. If P and Q are statements, under what conditions on the truth values of P and Q is $P \vee Q$ true?
20. If P and Q are statements, under what conditions on the truth values of P and Q is $P \wedge Q$ true?
21. If P and Q are statements, under what conditions on the truth values of P and Q is $P \Rightarrow Q$ true?
22. If P, Q and R are statements, under what conditions on the truth values of P, Q and R is $(P \Rightarrow Q) \Rightarrow R$ true?
23. If P, Q and R are statements, under what conditions on the truth values of P, Q and R is $P \Rightarrow (Q \Rightarrow R)$ true?