MATHEMATICS 300 — SPRING 1997

Introduction to Mathematical Reasoning

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Remarks on the number π

In order to do one of the homework problems, you needed to know lots of figures of the decimal expansion of the number π. So here is π with FOUR HUNDRED decimal figures, which is much much more than you need:

$$\begin{aligned}\pi \sim 3.&14159265358979323846264338327950288419716939937510582097494459230781640628 6\\&2089986280348253421170679821480865132823066470938446095505822317253594081 28\\&4811174502841027019385211055596446229489549303819644288109756659334461284 75\\&6482337867831652712019091456485669234603486104543266482133936072602491412 73\\&7245870066063155881748815209209628292540917153643678925903600113305305488 20\\&4665213841469519415116094\end{aligned}$$

Notice that each digit appears at least twice. Actually, all you need to make sure that each digit appears at least twice is to look at the first FIFTY (non rounded off) figures:

$$\pi \sim 3.14159265358979323846264338327950288419716939937510$$

Take a look at the above expansion and verify that each of the digits 0,1,2,3,4,5,6,7,8,9 appears at least twice.

Actually, the following is an *open problem*:

Is it true that every single one of the ten digits appears in π 10% of the time?

The precise meaning of this statement is as follows: for $k \in \{0,1,2,3,4,5,6,7,8,9\}$, $n \in \mathbf{N}$, define $f(k,n) =$ the number of times that k appears in the first n figures of the decimal expansion of π. (For example: $f(1,4) = 2$, because among the first 4 decimal figures of π the digit 1 occurs twice. Similarly, $f(0,49) = 1$, $f(0,50) = 2$, $f(3,40) = 6$, and so on.) Then the "proportion" of times that k occurs among the first n figures is $\frac{f(k,n)}{n}$. The question is whether "for large n" the proportion $\frac{f(k,n)}{n}$ is approximately 0.1. More precisely:

Is it true that $\lim_{n\to\infty} \frac{f(k,n)}{n} = 0.1$ for every $k \in \{0,1,2,3,4,5,6,7,8,9\}$?

Most mathematicians suspect that the answer is "yes," but so far nobody has been able to prove it.

Most students in the class used their calculator, which gave them a value of "π" with a few decimals, for example 3.1415926536. **This is not the number π!** This is just a rounded off approximate value of π! To answer the homework question this approximation was not sufficient, and you needed a much better one.

In past years, a few students have complained, with arguments such as "how was I supposed to know that you wanted me to get a better value of π?" or "why didn't you tell us what we had to do?" The answer is quite simple: in this problem you need π with 50 figures **because that's what is needed, not because I tell you. My personal wishes have nothing to do with it.** If I ask you to buy bread, then you do what is needed to buy bread. I don't have to tell you "you have to get up from your chair, you have to walk to the door, you have to open the door (if it isn't already open), you have to find out where there is a bakery or a supermarket, you have to find out how to go there, you have to go there, you have to buy the bread, etc." It goes without saying that you have to figure out what needs to be done and how to do it **not because I tell you but because that's what is needed.** Similarly, if you have a math question, you have to figure out what information you need in order to answer it, and then you what is needed to get the information. (For example, to get π with lots to decimals you go to the library and look for a book of mathematical tables, or you use some math software, or you ask somebody if you don't know what to do.)