MATHEMATICS 300 — FALL 1999

Introduction to Mathematical Reasoning

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HANDOUT NO. 3 *(September 16, 1999)*

READING FOR THE FIRST SIX DAYS OF CLASS

(that is, for Sept. 2-9-13-16-20-23)

- the book's "Preface for the Student,"
- the book's Chapter 1 (all of it),
- the instructor's Handouts Nos. 1, 2 and 3.

THE DEFINITION OF "INTERVAL"

The following definition of "interval" was given in class. It is much better than the one given in the book, so you should always be working with this definition. Later on, we will discuss why this definition is better.

DEFINITION. An *interval* is a subset S of $\mathbb{R}$ that satisfies the following property:

(I) for all $x, y \in S$, and all $t \in \mathbb{R}$, if $x < t < y$ then $t \in S$.

A REMARK ON DEFINITIONS

A *definition* is a statement that explains what a word or phrase means. In a definition, the word or phrase being defined is called the *definiendum*, and it should always be highlighted in some way, for example by underlining it, or putting it in some special font such as boldface or italics.

For example: if I write

An elephant is a four-legged animal.

then this is a correct statement, because what it says is true. But this is not a good definition of "elephant," because it does not explain to someone who does not know it already what the word "elephant" means. (A four-legged animal could also be a giraffe or a zebra, so this statement is not appropriate as a definition of "elephant," because it is not specific enough to single out the concept of "elephant" and distinguish it from that of any other animal.)

On the other hand, if I had written

An *elephant* is a four-legged animal.

then this would have been wrong, because by highlighting the word "elephant" I am saying that this is intended to be a definition of that word, and we know that, as a definition, this statement is not good enough.

YOU ARE STRONGLY ADVISED TO DO THE FOLLOWING THINGS:

- Look at the definition of "interval" given above, and see how it conforms to the specification that the definiendum must be highlighted.
- Look at al the definitions given in Chapter 1 of the book, figure out what the definiendum is in each case, and verify that the definition conforms to the specification that the definiendum must be highlighted. (Note: if you find any definition in the book where this rule is violated, please bring it to my attention, and I will give you some bonus credit points for your homework.)
- Write the definition of "interval" on your own—without copying it from this handout—as if you were answering the question "define 'interval' " in a quiz or exam, and then check what you wrote by comparing with the right definition. Do this repeatedly, as many times as may be needed, until you get it right.

HOMEWORK ASSIGNMENT NO. 3

This assignment is due on Thursday Sept. 23.

Before you start writing your homework, read carefully the parts of Handouts 1 and 2, on "writing mathematics." WARNING: having looked at Homework No. 1, I am under the impression that some students have not followed this advice, and have handed in assignments that will be marked "unaccepatble" and returned unread, exactly as was explained in Handout No. 1. If you think you may be one of those students, then please try to be more careful this time.

- Problems 1.31, 1.34, 1.36, 1.39, 1.41, 1.44.
- Determine the image of the function $f : \mathbb{N} \times \mathbb{N} \to \mathbb{Z}$ given by $f(n, m) = n^2 - m^2$.
- Determine the image of the function $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ given by $f(n, m) = |n^2 - m^2| + 1$.
- Suppose you had been asked: "Determine the image of the function $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ given by $f(n, m) = |n^2 - m^2|$." What would you say?