MATHEMATICS 300 — FALL 1998

Introduction to Mathematical Reasoning

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HANDOUT NO. 3

September 14, 1998

The **first midterm exam** is going to be on
WEDNESDAY, OCTOBER 14.

HOMEWORK ASSIGNMENT NO. 2.

The second homework assignment is due on **Wednesday, September 23**. It consists of:

a. Problem 3 in Page 16 of the book.
b. Problem 4 in Page 16 of the book, all the nonstarred parts,
c. Problem 5 in Page 16 of the book, all the nonstarred parts,
d. Problem 8 in Pages 16-17, all the nonstarred parts,
e. Problems 1, 4, 5, 6 in Pages 25-26, all the nonstarred parts.

plus

f. Problems 1,2,3,4,5 in Handout No. 4, pages 8 and 10.

READING FOR SEPTEMBER 14 AND 16.

Sections 1.3 and 1.4 of the book, and Handouts 3 and 4.

MORE REVIEW QUESTIONS.

Note: *the numbering of the review questions continues that of Handout No. 1.*

43. According to the instructor, as explained in Handout No. 3, why should a proof where one step is wrong be regarded as being totally wrong and not deserving partial credit?
44. In what way is a proof analogous to a chain?
45. According to the instructor, as explained in Handout No. 3, why is it necessary to have precise rules to determine what constitutes a proof?
46. Explain how theorem-proving and chess-playing resemble each other, and how they are different from each other.
47. List the symbols of a propositional calculus.
48. Give a precise definition of "well-formed formula" of a propositional calculus.

49. Why is it better to talk about **a** logical form of a statement or of an inference rather than of **the** logical form?
50. In general, is there such a thing as *the* propositional form of a statement, or can a statement have more than one propositional form?
51. Given an example of a statement having at least four different propositional forms, and then compare these forms and indicate for each pair of forms which one is coarser and which one is finer.
52. What is a "necessary condition"?
53. Indicate which of the following are good symbolic translations of "A is a necessary condition for B":

    1. $A \Longrightarrow B,$
    2. $B \Longrightarrow A\ ,$
    3. $A \Longrightarrow (\sim B)\ ,$
    4. $(\sim B) \Longrightarrow A\ ,$
    5. $(\sim A) \Longrightarrow B\ ,$
    6. $B \Longrightarrow (\sim A)\ ,$
    7. $(\sim A) \Longrightarrow (\sim B)\ ,$
    8. $(\sim B) \Longrightarrow (\sim A)\ .$

54. What is a valid inference?
55. What is a logically valid inference?
56. Is

$$\frac{\begin{array}{r}\text{My father is married.}\\ \text{My mother is married.}\end{array}}{\text{My father and my mother are married.}} \qquad (11)$$

    a logically valid inference?
57. For the simple formal language of §3 of Handout 4, give three examples of strings consisting of a's and b's that are well formed, and three that are not. Your strings should be at least 10 letters long. Justify your answers.
58. Explain what is wrong with the following argument: The equation $x^4 - 108x + 242$ does not have real solutions because it has imaginary solutions.