MATHEMATICS 300 — SPRING 2001

Introduction to Mathematical Reasoning (640 - 300, Section 03)

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HANDOUT NO. 2 *(February 1, 2001)*

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§1. HOMEWORK ASSIGNMENTS NOS. 2 AND 3

Assignment No. 2 was due today, February 1 (but Problem (N) can be handed in late). Assignment No. 3 is due on Thursday February 8.

ASSIGNMENT NO. 2:

- Section 2 of the book, Problems 1(b)(c), 4, 7.
- Section 3 of the book, Problems 2, 5, 6, 10, 12, 13.
- Section 4 of the book, Problems 3, 5, 6, 7, 9.
- PROBLEM (N): A student gave the following "definition of 'composite number' ": we say that a natural number n is *composite* if there exists a natural number b such that $1 < b < n$ and $n|b$. **Prove** that with this definition of "composite number" there do not exist any composite numbers.

ASSIGNMENT NO. 3:

- Section 5 of the book, Problems 1(a)(d)(e), 3, 4, 8, 11(b)(c)(e), 12(b), 18.
- Section 6 of the book, Problems 2, 6, 7, 9(d)(f)(g), 12, 15.
- Here are five sentences about giraffes, cows, sheep, Olivia, Dolly, and the binary relation "taller than."
  (1) Cows are taller than sheep. (That is, "every cow is taller than every sheep," or, if you prefer, "for every x and every y, if x is a cow and y is a sheep, then x is taller than y.")
  (2) Giraffes are taller than cows.
  (3) If something is taller than something else which is in turn taller than a third thing, then the first one is taller than the third one. (That is "for every x, every y and every z, if x taller than y and y is taller than z, then x is taller than z.")
  (4) Olivia is a giraffe.
  (5) Dolly is a sheep.

Using sentences (1), (2), (3), (4), (5), *and no other fact about giraffes, cows, sheep, Olivia, Dolly, and the binary relation "taller than,"* prove that Olivia is taller than Dolly.

- Here are four answers given by students to the question "express 30 as the difference of the square of two natural numbers":

  1. This cannot be done because in all the ways of expressing 30 as the difference of two numbers none of them hold.
  2. This cannot be done because there do not exist two natural numbers n and m such that $n^2 - m^2 = 30$.
  3. This cannot be done because 30 is divisible by 6, and a number that is divisible by 6 cannot be expressed as a difference of two squares of natural numbers.
  4. I have tried to express 30 as the difference of the squares of two natural numbers, but I wasn't able to figure out any way to do it. So I suspect that it cannot be done, but I do not know how to prove it.

  Discuss, criticize, and compare these four answers. (In your critical analysis of these answers, do not hesitate to be harsh if you think harshness is appropriate. But do not criticize anything without giving *good arguments* for your criticism.) Which one (or which ones) of the four should in your opinion get the highest grade? Which one (or which ones) of the four should in your opinion get the lowest grade?

- To answer the question "Prove that every year must have a Friday the 13th," a student wrote: "There are 52 weeks in a year. Let x be the number of Fridays in a year. Then $x = 52$. Since $x > 13$, there has to be at least a Friday the 13th in every year." Discuss and criticize this answer. (Hint: imagine that the Earth has slowed down a little bit so that the year has 364 days rather than 365; imagine that a new calendar—the "Sussmannian calendar"—has been adopted, in which the year is divided into 13 months rather than 12, each month having exactly 28 days. In the Sussmannian calendar there are 52 weeks in each year, and 52 is of course > 13. Would it still be true in the Sussmannian calendar that every year has a Friday the 13th? If not, why not? Does the student's purported "argument" apply to the Sussmannian calendar? If not, then why would it apply for the "true" calendar but not for the Sussmannian calendar? Of course, you may object that the Sussmannian calendar is just a figment of our imagination, and has nothing to do with reality, because for the true calendar the year has 365 days, not 364, there are 12 months, not 13, and the months do not have 28 days but have variable lengths (31, 28, 31, 30, 31, 30, 31, 31, 30, 31, 30 ,31). In other words, the real calendar is **not** the Sussmannian calendar. But, where in the "proof" does the author **use** the fact that the real calendar is not the Sussmannian calendar?)