MATHEMATICS 300 — FALL 1998

Introduction to Mathematical Reasoning

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HANDOUT NO. 15. November 30, 1998

THERE WILL BE ONE LAST HOMEWORK ASSIGNMENT (NO. 10) DUE ON WEDNESDAY, DECEMBER 9.

You will get the list of problems for this assignment on Wednesday, December 2.

Remember that:

1. The **second midterm test** is on Wednesday, December 2.
2. The **final exam** is on Wednesday, December 16, 8:00 pm to 11:00 pm.
3. During the week of December 7-11, we have **three** meetings, on Monday, Wednesday and Thursday, because Thursday Dec. 10 runs on a Monday schedule.

SOLUTIONS TO SOME HOMEWORK PROBLEMS

PROBLEM 1 OF ASSIGNMENT 9: Find integers a, b such that $37.a + 53.b = 1$.

SOLUTION: First divide 53 by 37, using the Euclidean algorithm. You get

$$53 = 1.37 + 16\,. \tag{1}$$

Then divide 37 by 16. You get:

$$37 = 2.16 + 5\,. \tag{2}$$

Next, divide 16 by 5. You get:

$$16 = 3.5 + 1\,. \tag{3}$$

Then (3) gives

$$1 = 16 - 3.5\,, \tag{4}$$

and (2) gives

$$5 = 37 - 2.16\,. \tag{5}$$

If we plug the expression for 5 given by (5) into (4), we get

$$1 = 16 - 3.(37 - 2.16) = (1 - 3.(-2)).16 + (-3).37\,,$$

so

$$1 = 7.16 + (-3).37\,. \tag{6}$$

Now (1) gives

$$16 = 53 - 37\,. \tag{7}$$

If we plug the expression for 16 given by (7) into (6), we get

$$1 = 7.(53 - 37) + (-3).37 = 7.53 + ((-7) + (-3)).37\,,$$

so

$$7.53 - 10.37 = 1\,.$$

So we can take $\boxed{a = 7,\ b = -10.}$

PROBLEM 3 OF ASSIGNMENT 9: Prove or disprove: there exist integers a, b such that $103.a + 107.b = 1$. If the numbers exists, find them.

SOLUTION: First divide 107 by 103, using the Euclidean algorithm. You get

$$107 = 1.103 + 4\,. \tag{1}$$

Then divide 103 by 4. You get:

$$103 = 25.4 + 3\,. \tag{2}$$

Next, divide 4 by 3. You get:

$$4 = 1.3 + 1\,. \tag{3}$$

Then (3) gives

$$1 = 4 - 3\,, \tag{4}$$

and (2) gives

$$3 = 103 - 25.4\,. \tag{5}$$

If we plug the expression for 3 given by (5) into (4), we get

$$1 = 4 - 3 = 4 - (103 - 25.4)\,,$$

so

$$1 = 26.4 - 103\,. \tag{6}$$

Now (1) gives

$$4 = 107 - 103\,. \tag{7}$$

If we plug the expression for 4 given by (7) into (6), we get

$$1 = 26.(107 - 103) - 103\,,$$

so

$26.107 - 27.103 = 1$. 2 So we can take $\boxed{a = 26,\ b = -27.}$