MATHEMATICS 300 — SPRING 1997

Introduction to Mathematical Reasoning

Instructor: H. J. Sussmann

HANDOUT NO. 11. March 13, 1997

READING ASSIGNMENTS SO FAR

- For the week of January 28 and 30, you have to read *all of Chapter 1.* Plus, of course, the first handout.
- For the week of February 4 and 6, you have to read *Sections 1, 2 and 3 of Chapter 2.*
- For the week of February 11 and 13, you have to read *Sections 2.4, 3.1 and 3.2.*
- For the week of February 18 and 20, you have to read *Sections 3.3 and 3.4.*
- No new reading assignment for the week of February 25 and 27.
- For the weeks of March 4-6 and 11-13, you have to read *Sections 3.4 and 3.5.*
- For the week of March 25-27, you have to read *Handout No. 10* (very carefully) and the article by P. R. Halmos.

HOMEWORK ASSIGNMENTS SO FAR

(1) *Originally due on Tuesday, January 28. The deadline was subsequently extended to February 4.* Page 14, Problem 4; Page 15, Problem 7; Page 26, Problem 10; Page 27, Problem 11; Page 37, Problems 2 and 8.

(2) *Due Tuesday, February 4.* Page 14, Problem 5; Page 47, Problems 1 (parts (a), (j), (q)), 2, and 3; Page 48, Problems 5 (parts (a), (c)) and 7; Page 49, Problems 11 and 12.

(3) *Due Tuesday, February 11.* Page 56, Problem 1, parts (a), (b), (f), (g), (h), (j), (l). Problem 4, parts (a), (b), (c), (e), (h), (i), (j). Page 60-61, Problem 2. Page 61, Problems 3,4. And, in addition, the following:
  (A) Let S be the set of all words consisting of exactly three vowels and three consonants, with repetition allowed for the vowels but not for the consonants. (For example: *arabap*, *benito*, *yaaexz* and *xaeiyz* belong to S, but *bavenii*, *baeiou*, *babeic* and *gugugu* do not. Recall that in the English alphabet there are 5 vowels and 21 consonants.)
  (i) Determine how many members S has.
  (ii) Determine what percentage of the words in S contain the letter "a". Is it higher or lower than 50%?

(4) *Due Tuesday, February 18.* Page 70, problems 4, 6, 8, 9. Page 72, problem 14. Page 73, problem 16. Page 76, problems 6 and 9.

(5) **Optional homework assignments**, *due Tuesday, February 25.* See handout No.7 for details.

(6) *Due Thursday, March 6.* Page 90, Problems 6, 9, 10. Page 99, Problems 1, 2. Page 100, Problems 5, 6. Page 121, Problems 8, 9.

(7) *Due Thursday, March 13.* Pages 122-123, Problems 11, 12, 16, 17, 18, 21, 22.

(8) *Due Tuesday, April 1.* (a) Problems 1, 2, 3, 4, 5, 6 of Handout No. 10. (b) For each of the four statements (S1), (S2), (S3), (S4) below, decide whether or not the statement is true. In each case, I want you to give a **proof**, i. e. a **convincing argument**, showing that the statement is true, or showing that it is false. I am not telling you which statements are true and which ones are false. You have to figure this out first. But, once you are convinced that a statement is true, you have to be able to convince me. Or, if you are convinced that it is false, then you have to convince me. Juts saying "I am sure it's true" or "I am sure it's false" will not do. I am **not** asking you to give a formal proof of your answer, using the rules and nothing else. (This can be done, but it would be a lot of work to write it out in full.)

Here are the four statements:

(S1) *There exists one and only one positive integer p with the property that the three numbers p, $p+2$ and $p+4$ are prime.*

(S2) *Every positive integer greater than 1 can be expressed as the sum of the squares of two positive integers.* (That is: $(\forall n \in \mathbf{N})\Big(n > 1 \rightarrow (\exists m \in \mathbf{N})(\exists p \in \mathbf{N})(n = m^2 + p^2)\Big)$.)

(S3) *The average of two positive real numbers is always greater than or equal to the square root of their product.* (That is: $(\forall x \in \mathbf{R})(\forall y \in \mathbf{R})\Big((x > 0 \wedge y > 0) \rightarrow \frac{x+y}{2} \geq \sqrt{xy}\Big)$.)

(S4) *The intersection of two intervals is an interval.* (That is,

$$(\forall I)(\forall J)\left(\Big((I \text{ is an interval}) \wedge (J \text{ is an interval})\Big) \rightarrow (I \cap J \text{ is an interval})\right).$$

Naturally, you will have to use the definition of "interval.")