MATHEMATICS 300 — SPRING 2001

Introduction to Mathematical Reasoning (640 - 300, Section 03)

H. J. Sussmann

HANDOUT NO. 1 *(January 18, 2001)*

§1. INFORMATION ON THE COURSE

Course schedule. We meet on **Mondays and Thursdays, 3rd period (11:30 to 12:50)**, in Hickman 115 (at Douglass College).

About the instructor. My *name* is **H. J. Sussmann.** My *office* is **Hill 704**. The *phone extension* is 5-5407. The *e-mail address* is **sussmann@math.rutgers.edu**.

Web page. I have set up a *Web page* for our Math 300 section:

http://www.math.rutgers.edu/~sussmann/math300page.html

All the handouts, including this one, will be available there.

Office hours. My office hours will be:

- Thursday, fourth and fifth period, in **Room 207, Chemistry Building, Douglass Campus**;
- any other time, by appointment, in my office, **Hill 704**.

You should let me know that you are coming to see me *even if you are planning to do so during my regular office hours*. REASON: I don't want to have to stay in my office waiting just in case someone might show up. If I know that you are coming then I will be there, but if you don't tell me that I should expect you then I will feel free to go away and do other things.

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§2. HOMEWORK, MIDTERMS, ETC.

Homework will count for about one third of your grade. There will be *two midterms*, which will count—together—for another third of your grade. The *final exam* will count for the remaining third.

Please make sure to read all the handouts carefully, because they contain important information. For example, this handout contains lots of things you need to know in order to do your homework.

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§3. THE TEXTBOOK

We will be using:

- the book *Mathematics, a discrete introduction*, by **Edward R. Scheinerman** (Brooks/Cole Publishing Co., Pacific Grove, CA, 2000),
- the handouts written by the instructor.

The material of the handouts is an integral part of the course, as much as that of the book.

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§4. READING FOR THE FIRST 1&1/2 WEEK
(that is, for January 18, 22 and 25)

- the book's note "To the Student,"
- the book's Chapter 1,
- the instructor's Handout No. 1.

The instructor's Handout No. 2 may be added to this list, if it becomes available on time.

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§5. SOME IMPORTANT REMARKS

REMARK NO. 1: The "natural numbers"—also called "whole numbers," or "strictly positive integers"—are $0, 1, 2, 3, \ldots$. In particular, **the number** 0 **is a natural number.** ♢

REMARK NO. 2: Later on, after we have started to work with set notation, we will use $\mathbb{N}$ as a name for the set of all natural numbers, so

$$\mathbb{N} = \{\, 0, 1, 2, 3, 4, \ldots \}\,.$$

The members of $\mathbb{N}$—that is the natural numbers— are also called **nonnegative integers**.

We will use $\mathbb{Z}$ to denote the set of **all** integers, that is,

$$\mathbb{Z} = \{\, 0, 1, -1, 2, -2, 3, -3, 4, -4, \ldots \}\,.$$

A real number (and, in particular, an integer) is "positive" if it is > 0. A number is "nonnegative" if it is ≥ 0.

In particular, 0 **is not positive, but it is nonnegative.** 0 **is a nonnegative integer, but** 0 **is not a positive integer.**

The **positive integers** are $1, 2, 3, 4, \ldots$. ♢

REMARK NO. 3: **In this course, whenever you are given a problem where you are asked to do something, your answer should be either:**

(a) to do what you were asked to do,

or

(b) to show—that is *prove*—that it cannot be done.

(See, for example, Problems 2 and 4 below, where the correct answer is "what you asked me to do cannot be done.")

Notice that, when the answer is that "it cannot be done," it is not enough for you to *say* that it cannot be done. *You have to tell me why.* In other words, you have to *prove* that it cannot be done.

This remark is very important, and will apply throughout the semester, not just during the first week. So please read it and reread it until you are sure you got the point. ♢

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§6. SOME EXAMPLES OF PROBLEMS WITH CORRECT SOLUTIONS

Here are some problems with correct solutions:

PROBLEM 1: Express the number 26 as a sum of two odd natural numbers.

SOLUTION: $26 = 3 + 23$. ♢

REMARK: *There are lots of other solutions, of course! For example,* $26 = 7 + 19$, $26 = 13 + 13$.

FOOD FOR THOUGHT: *How many solutions does this problem have?* Before you answer this, make sure you give a precise definition of "solution" (that is, tell us what you mean by "solution"). This is important, because for this problem there is more than one reasonable way to define what you mean by "solution," and the choice of definition makes a difference. People who define "solution" differently may come up with different answers!

PROBLEM 2: Express the number 27 as a sum of two odd natural numbers.

SOLUTION: This is impossible. REASON: the sum of two odd numbers is always even. Since 27 is odd, it cannot be the sum of two odd numbers. ♢

PROBLEM 3: Prove that $2 + 2 = 5$.

SOLUTION: This cannot be done. REASON: The statement "$2 + 2 = 5$" is false, and false statements cannot be proved.

PROBLEM 4: Prove that for every positive integer x there is another positive integer y such that $0 < y < x$.

SOLUTION: This cannot be done. REASON: The statement that we are being asked to prove is false, and a false statement cannot be proved.

The reason that the statement is false is as follows: 1 is a positive integer, and there is no positive integer y such that $0 < y < 1$, because 1 is the smallest positive integer. So if we take x to be 1 then the conclusion is not true. So it is not true that "for *every* positive integer x there is another positive integer y such that $0 < y < x$." ♢

PROBLEM 5: Express the number 12 as the difference of the squares of two natural numbers.

SOLUTION: $12 = 16 - 4$, so $12 = 4^2 - 2^2$. ♢

PROBLEM 6: Express the number 20 as the sum of the squares of two natural numbers.

SOLUTION: $20 = 4 + 16 = 2^2 + 4^2$. ♢

PROBLEM 7: Express the number 4 as the sum of the squares of two positive integers.

SOLUTION: This cannot be done. REASON: here are all the ways to express 4 as a sum of two positive integers:

$$4 = 1 + 3\,, \qquad 4 = 2 + 2\,.$$

In neither of these two equalities are the two terms on the right-hand side squares.

PROBLEM 8: Express the number 4 as the sum of the squares of two natural numbers.

SOLUTION: $4 = 0 + 4 = 0^2 + 2^2$. ♢

PROBLEM 9: Express the number 12 as the sum of the squares of two natural numbers.

SOLUTION: This cannot be done. REASON: here are all the ways to express 12 as a sum of two natural numbers:

$$12 = 0 + 12\,,\ \ 12 = 1 + 11\,,\ \ 12 = 2 + 10\,,\ \ 12 = 3 + 9\,,\ \ 12 = 4 + 8\,,\ \ 12 = 5 + 7\,,\ \ 12 = 6 + 6\,.$$

In none of these expressions are the two terms on the right-hand side squares. ♢

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§7. HOMEWORK ASSIGNMENT NO. 1

This assignment is due on Thursday, January 25.

Before you start writing your homework, read carefully the rest of this handout, in particular §5 and §6, and §8, on "writing mathematics."

Don't worry if some of these problems seem too hard. Just do as much as you can. One of the main purposes of this set of questions is to get you to experience what it is like to think about problems that could be quite hard (even if they look easy!), so that you will appreciate what it means to find a **truly good solution**. *For you to see this, you have to think about the problems first, and if you cannot solve them* **after you have tried to** *and then you see the solution you will understand much better what a* **real proof** *is.*

- Problems 3, 4, 5, 6 in page 5 of the book.
- Prove that every year must have a Friday the 13th.
- Express the number 28 as the difference of the squares of two natural numbers.
- Express the number 29 as the difference of the squares of two natural numbers.
- Express the number 30 as the difference of the squares of two natural numbers.
- (*Optional. This could be quite hard! Do not worry if you cannot do it, but at least try to* **think** *about it.*) Express the number $4,582,623$ as the sum of the squares of two natural numbers.

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§8. WRITING MATHEMATICS

You should write so that you can be easily understood by a properly trained English-speaking individual. In particular, this means that you must

- Use *complete English sentences*, that make clearly identifiable *statements* with a *clear meaning* that can can be understood by anyone reading what you wrote. For example:
  - If you tell me that "she is very smart," but you haven't told me who "she" is, then I don't know what you are talking about, so you haven't made a statement with a clear meaning.

  - If you write "$x > 0$," but you haven't told me who "x" is, then I don't know what you are talking about, so you haven't made a statement.
  - If I ask you to state Pythagoras' theorem and you answer "$a^2 + b^2 = c^2$," then nobody will know what you are talking about, because you have not said what "a," "b," and "c" are supposed to be.

- Avoid exaggerated or incorrect use of cryptic mathematical notation.
- Explain what you are doing.
- Make sure that letter "variables" are only used if either: (i) it has been said before what these letters stand for, or (ii) they are "closed variables" (or "dummy variables," or "bound variables") in the sense that will be discussed in detail in class, and also in subsequent handouts.
- Provide proper connectives between equations as well as between ideas.
- Make sure that all the rules of English grammar (including those of spelling and punctuation) are strictly obeyed.
- Try to say things correctly, following all the rules, but *in your own words.* Please *no rote learning.* If you have to memorize a definition or a statement, then this is an indication of lack of understanding.
- Please proofread carefully what you hand in. Ideally, you should read and reread and revise almost any formal communication. **Neatness and clarity count**, as you well know if you've tried to read any complicated document.
- *Do not assume that the people reading your paper can read your mind. Do assume that they are intelligent, but also assume that they are busy, and cannot and will not spend an excessive amount of time puzzling out your meaning. Communication is difficult, and written technical communication is close to an art.*

Effective written exposition will be worth at least 50% of your grade. Conversely, bad or unclear exposition may be penalized as much as 50% of the grade or even more.

- The best reference known to me on effective writing is *The Elements of Style* by Strunk and White, a very thin paperback published by Macmillan. It isn't expensive, and it is easy to read. I recommend it.

Pay attention to presentation, especially for the homework:

- A nicely typed homework (e.g., using a word processor) is preferable to handwritten work. Handwritten work is acceptable too, but in that case:
  - If you have to cross out lots of words, then you should rewrite the whole thing anew, **cleanly and neatly**. If you are not willing to spend some of your time doing this; if what you hand in shows that you were in a hurry and that you did not make the effort to write things neatly and properly, then there is no reason for me to spend any of my time reading what you wrote, and I will not do it.

  - You should use a pen. Never use a pencil.
  - Use any color other than red (for example, black, blue, or green), but DO NOT USE RED. (Reason: The use of red is reserved for the instructor's comments.)

- Make sure that **your name appears in every sheet of paper you hand in**, and that if you are handing in more than one sheet then the sheets are **stapled** and the pages are **numbered**.
- If you tear off the sheets from a spiral notebook, please make sure before you hand them in that there are none of those ugly hanging dshreds of paper at the margins. Use scissors, or a cutter, if necessary.

Homework assignments that fail to meet the above requirements will be marked "unacceptable" and returned unread.

Keep in mind that

If you hand in a homework assignment that

- **is written carelessly or in a hurry,**

or

- **has lots of words crossed out,**

or

- **has unreadable handwriting,**

or

- **has pages that are unstapled or unnumbered or fail to show your name,**

or

- **has shreds of paper at the margins**

(in other words, anything that looks ugly, or anything that you would be ashamed to hand in, say, if you were writing a letter to apply for a job), then

- **this is a form of disrespect towards your instructor, and that's exactly how you should expect that your instructor will treat it.**