MATHEMATICS 300 — FALL 1999

Introduction to Mathematical Reasoning

H. J. Sussmann

HANDOUT NO. 1 *(Revised version, September 9, 1999)*

INFORMATION ON THE COURSE

Course schedule. We meet on **Mondays and Thursdays, 4th period**, in ARC 205.

About the instructor. My *name* is **H. J. Sussmann.** My *office* is **Hill 704**. The *phone extension* is 5-5407. The *e-mail address* is **sussmann@math.rutgers.edu**.

Web page. I have set up a *Web page* for our section of Math 300:

http://www.math.rutgers.edu/~sussmann/math300page.html

All the handouts, including this one, will be available there.

Office hours. My office hours will be:

- Monday, fifth period, and Thursday, fourth period;
- any other time, by appointment.

You should let me know that you are coming to see me *even if you are planning to do so during my regular office hours.* REASON: I don't want to have to stay in my office waiting just in case someone might show up. If I know that you are coming then I will be there, but if you don't tell me that I should expect you then I will feel free to go away and do other things.

HOMEWORK, MIDTERMS, ETC.

Homework will count for about one third of your grade. There will be *two midterms*, which will count—together—for another third of your grade. The *final exam* will count for the remaining third.

Please make sure to read all the handouts carefully, because they contain important information. For example, this handout contains lots of things you need to know in order to do your homework.

THE TEXTBOOK

We will be following:

- the book *Mathematical Thinking: Problem-solving and proofs*, by **John P. D'Angelo** and **Douglas B. West**, Second Edition, 1999, pre-publication version. (This edition can be purchased at the RU bookstore.)
- the handouts written by the instructor.

The material of the handouts is an integral part of the course, as much as that of the book.

READING FOR THE FIRST WEEK (that is, for Sept. 2-9)

- the book's "Preface for the Student,"
- the book's Part I, Chapter 1, first four sections (that is, up to and including the section on "functions");
- the instructor's Handout No. 1.

A REMARK ON PROBLEMS AND SOLUTIONS

In this course, whenever you are given a problem where you are asked to do something, your answer should be either: (a) to do it, or (b) to show—that is *prove*—that it cannot be done.

For example, here are some problems with correct solutions:

PROBLEM: Express the number 26 as a sum of two odd natural numbers.

SOLUTION: $26 = 3 + 23$. $\diamondsuit$

PROBLEM: Express the number 27 as a sum of two odd natural numbers.

SOLUTION: This is impossible. REASON: the sum of two odd numbers is always even. Since 27 is odd, it cannot be the sum of two odd numbers. $\diamondsuit$

PROBLEM: Prove that for every positive real number x there is another real number y such that $0 < y < x$. (Note[1]: "positive" means "> 0.")

SOLUTION: Let x be a real number such that $x > 0$. Take $y = \frac{x}{2}$. Then $y > 0$. On the other hand, $y < x$, because $y + y = x$ and $y > 0$, so $y = x - y < x$. Therefore $0 < y < x$. $\diamondsuit$

PROBLEM: Prove that for every natural number x there is another natural number y such that $0 < y < x$. (Note: here too, "positive" means > 0." The "natural numbers"—also called "whole numbers"—are $1, 2, 3, \ldots$)

SOLUTION: This cannot be done. REASON: The statement that we are being asked to prove is false, and a false statement cannot be proved.

The reason that the statement is false is as follows: 1 is a natural number, and there is no natural number y such that $0 < y < 1$, because 1 is the smallest natural number. So if we take x to be 1 then the conclusion is not true. So it is not true that "for *all* natural numbers x there is another natural number y such that $0 < y < x$." $\diamondsuit$

PROBLEM: Express the number 12 as the difference of the squares of two natural numbers.

SOLUTION: $12 = 16 - 4$, so $12 = 4^2 - 2^2$. $\diamondsuit$

[1] A QUESTION FOR THE STUDENT: Why did I feel it was necessary to have this note here?

PROBLEM: Express the number 12 as the sum of the squares of two natural numbers.

SOLUTION: This cannot be done. REASON: here are all the ways to express 12 as a sum of two natural numbers:

$$12 = 1 + 11\,,\ \ 12 = 2 + 10\,,\ \ 12 = 3 + 9\,,\ \ 12 = 4 + 8\,,\ \ 12 = 5 + 7\,,\ \ 12 = 6 + 6\,.$$

In none of these expressions are the two terms on the right-hand side squares. $\diamondsuit$

HOMEWORK ASSIGNMENT NO. 1

This assignment is due on Thursday Sept. 9.

Before you start writing your homework, read carefully the last section of this handout, on "writing mathematics."

Don't worry if some of these problems sem too hard. Just do as much as you can.

- Problems 12, 13, 14 and 24 in the list of problems of the book's "Preface for the Student";
- Express the number 28 as the difference of the squares of two natural numbers.
- Express the number 29 as the difference of the squares of two natural numbers.
- Express the number 30 as the difference of the squares of two natural numbers.

WRITING MATHEMATICS

You should write so that you can be easily understood by a properly trained English-speaking individual. In particular, this means that you must

- Use *complete English sentences*, that make clearly identifiable *statements* with a *clear meaning* that can can be understood by anyone reading what you wrote. (For example, if you tell me that "she is very smart," but you haven't told me who "she" is, then I don't know what you are talking about, so you haven't made a statement. Similarly, if you say "$x > 0$," but you haven't told me who "x" is, then I don't know what you are talking about, so you haven't made a statement. If I ask you to state Pythagoras' theorem and you answer "$a^2 + b^2 = c^2$," then nobody will know what you are talking about, because you have not said what "a," "b," and "c" are supposed to be.)
- Avoid exaggerated or incorrect use of cryptic mathematical notation.
- Explain what you are doing.
- Make sure that letter "variables" are only used if either: (i) it has been said before what these letters stand for, or (ii) they are "closed variables" (or "dummy variables," or "bound variables") in the sense that will be discussed in detail in class, and also in subsequent handouts.
- Provide proper connectives between equations as well as between ideas.

- Make sure that all the rules of English grammar (including those of spelling and punctuation) are strictly obeyed.
- Try to say things correctly, following all the rules, but *in your own words.* Please *no rote learning.* If you have to memorize a definition or a statement, then this is an indication of lack of understanding.
- **Pay attention to presentation**:
  - If you have to cross out lots of words, then you should rewrite the whole thing anew, **cleanly and neatly**. If you are not willing to spend some of your time doing this; if what you hand in shows that you were in a hurry and that you did not make the effort to write things neatly and properly, then there is no reason for me or the grader to spend any of our time reading what you wrote, and we will not do it.
  - Make sure that **your name appears in every sheet of paper you hand in**, and that if you are handing in more that one sheet then the sheets are **stapled** and the pages are **numbered**.
  - **Homework assignments that fail to meet these requirements will be marked "unacceptable" and returned unread.**

Do not assume that the people reading your paper can read your mind. Do assume that they are intelligent, but also assume that they are busy, and cannot and will not spend an excessive amount of time puzzling out your meaning. Communication is difficult, and written technical communication is close to an art.

Effective written exposition will be worth at least 50% of your grade. Conversely, bad or unclear exposition may be penalized as much as 50% of the grade or even more.

Please proofread carefully what you hand in. Ideally, you should read and reread and revise almost any formal communication. **Neatness and clarity count**, as you well know if you've tried to read any complicated document.

The best reference known to me on effective writing is *The Elements of Style* by Strunk and White, a very thin paperback published by Macmillan. It isn't expensive, and it is easy to read. I recommend it.