MATHEMATICS 300 — FALL 2001

Introduction to Mathematical Reasoning

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SOME EXAMPLES OF MISTAKES STUDENTS MADE IN THE SEPTEMBER 19 QUIZ

1. QUESTION: define "odd." ANSWER: Let x, y be integers. We say that x is odd provided that $x = 2y + 1$.

   WHAT IS WRONG WITH THIS? Let us test the definition proposed by the student in a concrete example. Let us take $x = 9$, $y = 7$. According to the student, "x is odd provided that $x = 2y + 1$." Let us plug in our x and our y. We get "9 is odd provided that $9 = 2 \cdot 7 + 1$." But $2 \cdot 7 + 1 = 15$, which is not equal to 9. So **according to this student's definition of "odd," the number 9 is NOT odd.** This is obviously not what we want, so the definition is wrong.

   GRADE: on a scale from 0 to 5, this gets a 0.

2. QUESTION: define "odd." ANSWER: Let x be an integer. We say that x is odd provided that $x = 2y + 1$.

   WHAT IS WRONG WITH THIS? Let us test the definition proposed. Take $x = 7$. Is it odd? To find out, we have to find out if it is true that $x = 2y + 1$. But there is no way on Earth that we can find out, because the author has not told us what y is. So **as stated, this definition is useless, because it requires that we use a mysterious y that we have no idea what it is.**

   GRADE: on a scale from 0 to 5, this gets a 0.

3. QUESTION: define "odd." ANSWER: Let x be an integer. We say that x is odd provided that $x = 2y + 1$ for some number y.

   WHAT IS WRONG WITH THIS? Let us test the definition proposed. Take $x = 6$. Is it odd? To find out, we have to find out if it is true that there is a number y such that $x = 2y + 1$. And, indeed, such a number exists, since we can take $y = \frac{5}{2}$. So **according to this student's definition of "odd," the number 6 IS odd.** This is obviously not what we want, so the definition is wrong.

   GRADE: on a scale from 0 to 5, this gets a 0.

4. QUESTION: define “odd.” ANSWER: Let x be an integer. We say that x is odd provided that $x = 2y + 1$ for some integer y.

   WHAT IS WRONG WITH THIS? This definition is almost perfect. Only one important thing is missing. **When you write a definition of a word or phrase, the word or phrase being defined must be highlighted, e.g. by underlining.** This is explained in detail in Remark 2, page 36 of the first set of instructor’s notes. I suggest you look at the definitions in the book. You will see that *the book always obeys this rule*. You should obey it too.

   GRADE: on a scale from 0 to 5, this gets a 4.

5. QUESTION: prove that 13 is odd. ANSWER: By the definition of “odd,” $13 = 2a + 1$. Solving the equation, we get $a = 6$. So 13 is odd.

   WHAT IS WRONG WITH THIS? First of all, the author is not telling us what the letter a is supposed to stand for.

   Second, and much more serious, the author seems to *deduce* the fact that $13 = 2a + 1$ from the fact that 13 is odd. But this is not permitted, because **at the point where the author says “By the definition of ‘odd,’ $13 = 2a + 1$” we do not know yet that 13 is odd, so we cannot use this fact to deduce anything from it.** The statement that 13 is odd is what we are trying to prove, and you cannot use in a proof the thing that you are trying to prove.

   GRADE: on a scale from 0 to 3, this gets a 1/2 or at most a 1.