MATHEMATICS 300 — FALL 2001

Introduction to Mathematical Reasoning

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1 The first midterm

The date of the first midterm has been changed. It is going to be on *Monday, October 22* (rather than on October 15 as originally scheduled).

2 Homework assignment due on Monday, October 22

Since the first test is on October 22, you may want to have your homework graded before that day. If you want to do that, then you can do the following:

1. hand in the assignment on Wednesday October 17 in class, or bring it to the Hill Center (and put it in my mailbox in the Math Dept mailroom in the 3rd floor of the Hill Center) on Thursday or Friday[1].
2. make sure you write your e-mail address *very clearly* on the first page;
3. keep a copy for yourself.

If you do this, then I will read your proofs as soon as I can and send you my comments by e-mail.

And here is the assignment due on October 22. It consists of **two problems**.

1. For each of the following four statements, give a completely formal proof, or show that the statement cannot be proved:

   (1) $\Big(\forall x, (x \text{ is a borogove} \wedge x \text{ is mimsy})\Big) \longleftrightarrow \Big((\forall x, x \text{ is a borogove}) \wedge (\forall x, x \text{ is mimsy})\Big)$

   (2) $\Big(\exists x, (x \text{ is a borogove} \wedge x \text{ is mimsy})\Big) \longleftrightarrow \Big((\exists x, x \text{ is a borogove}) \wedge (\exists x, x \text{ is mimsy})\Big)$

   (3) $\Big(\forall x, (x \text{ is a borogove} \vee x \text{ is mimsy})\Big) \longleftrightarrow \Big((\forall x, x \text{ is a borogove}) \vee (\forall x, x \text{ is mimsy})\Big)$

   (4) $\Big(\exists x, (x \text{ is a borogove} \vee x \text{ is mimsy})\Big) \longleftrightarrow \Big((\exists x, x \text{ is a borogove}) \vee (\exists x, x \text{ is mimsy})\Big)$

   REMARK: You should expect these proofs to involve aproximately 20 to 30 steps for each statement that can be proved.

[1] I also give you another option: you can e-mail your assignment to me as a *Postscript file*. But please do not e-mail your assignment to me in any way other than as a Postscript file.

2. Give a completely formal proof of the statement

$$(5)\ \forall X, \forall Y, \forall Z, \Big(\big((X \text{ is a set} \wedge Y \text{ is a set}) \wedge Z \text{ is a set}\big) \rightarrow \big((X \subseteq Y \wedge Y \subseteq Z) \rightarrow X \subseteq Z\big)\Big)$$

using the definition of set inclusion:

$$(6) \qquad \forall X, \forall Y, \big((X \text{ is a set} \wedge Y \text{ is a set}) \rightarrow (X \subseteq Y \leftrightarrow \forall x, (x \in X \rightarrow x \in Y))\big).$$

Note: for Problem 2, I tried writing a formal proof of (5) without working very hard to minimize the number of steps, and it took me 50 steps. You can probably do better than that.

Whoever writes the shortest proof of (5) will get a bonus, in the form of double credit for this entire assignment.

Note: if there is a tie for the smallest number of steps, the bonus points will be divided among all winners. It follows that **it is not in your interest to share your work with somebody else.**

Please make sure that your proofs are complete, and that each step is properly justified. The justifications should be written as in the examples of formal proofs you already have in the instructor's notes: the justification of each step should indicate how the step follows from previous steps according to the rules, or that it is a "preparation" step of the form "Assume A" or "Let a be arbitrary," or that it is a definition or an axiom. (In this homework there aren't any axioms yet[2]. These will come later in the course.)

Also, make sure that you indicate at each step the "level of proof" you are at. You can do this using variable indentation or numbers for proof levels, as you prefer. You have examples of both in the instructor's notes, Part 2.

[2]Except for the fact that, if you wish, you might want to regard Formula (6) as an axiom rather than a definition