MATHEMATICS 642:577 — Fall 2001

Mathematical Topics in Systems Theory

Instructor: H. J. Sussmann

CALCULUS OF VARIATIONS AND OPTIMAL CONTROL

The course will present a self-contained introduction to the classical calculus of variations and optimal control for problems involving curves. We will first discuss the statements of various necessary conditions for an optimum, starting with the Euler-Lagrange equation and then moving on to the Legendre and Weierstrass necessary conditions, and the Pontryagin Maximum Principle in both its classical version—for continuously differentiable maps—and more recent versions that assume a lot less smoothness. Examples of applications to various optimal control problems (including classical ones such as Johann Bernoulli's brachystochrone, as well as more modern ones such as the problems of Zermelo, Fuller, and Markov-Dubins-Reeds-Shepp) will be discussed, to show how the techniques are actually used to get concrete information about the solutions. All this will be done before we give the proofs of the various necessary conditions, because (a) the goal of this first part of the course will be to acquaint the students with the statements and how they are used, and (b) there is no need to spend a lot of time doing proofs of partial results that will be reproved later in the course in a more general setting.

Once we get to the most general statement currently[1] available, we will shift gears and present a detailed proof, based on the use of suitable theories of generalized differentials. This part will be preceded by an introduction to the basic concepts of nonsmooth analysis needed for the proof, and in particular to J. Warga's theory of derivate containers of Lipschitz maps and its extension, the theory of semidifferentials.

We will then resume the discussion of examples and of the use of the necessary conditions by showing how the natural language for translating the consequences of the necessary conditions into useful information is that of differential geometry, especially when systematic use is made of intrinsic, covariant objects such as Lie brackets of vector fields. This will be done by discussing various "intrinsic" ways to restate the Maximum Principle and using them to prove theroems about the qualitative properties of solutions.

Finally, if time permits, we will discuss sufficient conditions, fields of extremals, and regular synthesis.

The material to be presented in the course will combine the contents of several papers that are available in the instructor's Web page, http://www.math.rutgers.edu/~sussmann (click on "Papers available on the Web"), especially Papers No. 63, 62, 61, 58, 57, 53, 52, 51, 50, 49, 42, and 40.

[1]In this rapidly evolving area, "currently" means "at the time the course is taught." The most general statement available at that time may be more general than the one available now, in May 2001.