The Control Handbook. *By William S. Levine, Editor.* CRC Press, Inc., Boca Raton, 1996. \$145. xvi+1548 pp., hardcover. ISBN 0-8493-8570-9.

Control theory is a rich and varied field of science, within which one finds theoretical work of a mathematical nature involving a broad spectrum of techniques, such as linear algebra, function theory, ordinary and partial differential equations, functional analysis, differential geometry, algebraic geometry, stochastic analysis and combinatorics. This mathematical work marks one end of a continuous spectrum that, at the opposite extreme, contains applications to such things as "regulating the amount of water in a toilet tank, controlling the flow and generation of electrical power over huge geographic regions, regulating the behavior of gasoline engines, controlling the thickness of rolled products as varied as paper and sheet steel, and hundreds of controllers hidden in consumer products of all kinds," as the editor of *The Control Handbook* explains in his preface. Practitioners of the art and science of control theory work mostly in engineering departments, although in recent years there have been signs that the mathematical community may be starting to appreciate the beauty of the subject and its potential as an area for active involvement by mathematicians. The size of the field can be measured by the fact that the membership of the Control Systems Society —one of the professional societies within the Institute of Electrical and Electronic Engineers— is about half that of the entire American Mathematical Society. The depth and importance of control theory, both as a scientific field and as a key area of technology, is sometimes underappreciated by the general public, to whom computers with blinking lights often appear to be much more real than abstract scientific theories.

Mathematicians sometimes find it difficult to communicate with control engineers and to understand the problems that their engineering colleagues are trying to solve, because control questions are often *design* problems, in which one isn't asking whether some statement is true or false, but *how something should be done.*

Here is a simple example: suppose you have a system governed by a second-order differential equation $\ddot{x} + c\dot{x} + kx = u$, where u is an external force or "control" that can be chosen by the controlling agent. Suppose you can measure position but not velocity. (That is, you have direct access to x but not to $\dot{x}$.) Suppose your goal is to control the system by choosing u so that all the trajectories of the resulting "controlled dynamics" will go to zero as $t \to +\infty$, but *you don't know the values (or even the signs!) of the real numbers c and k.* Can the goal be attained? And if so how? An engineer, trying to explain this to a mathematician, might say something like "I want to choose u, depending on x but not on $\dot{x}$, c or k, in such a way that, no matter what the values of c and k are, the solution $t \to x(t)$ will go to zero as $t \to +\infty$, for all possible initial conditions." The mathematician, after some thinking, will probably answer as follows: "Theorem: it cannot be done. Proof: If a function $x \to u(x)$ has the properties you want, then you should in particular be able to choose $c = k = 0$. In that case, making the reasonable assumption that u is continuous, and letting $U(x) = \int_0^x u(s)\,ds$, it is easy to verify that the function $V(x, \dot{x}) = \frac{\dot{x}^2}{2} - U(x)$ is constant along all trajectories, so if you start with an initial condition $x(0) = \alpha$, $\dot{x}(0) = \beta$, such that $V(\alpha, \beta) \neq 0$, then the solution cannot go to zero." This, however, will not satisfy the engineer, who will most certainly reply: "I never said that u had to be a *function* of x in the usual mathematical sense. It's true that I want u to 'depend on x,' but I see no reason for not allowing u at a time t to depend on *past values* of x as well as current values. So, if we end up choosing u at time t as a *functional* of the whole past of x up to time t, rather than as a function of the value of x at time t, that would still be acceptable." In fact, it turns out that our engineer's intuition was right, and our mathematician's impossibility proof is wrong: one *can* construct an "adaptive controller" that does the job, as long as one allows a choice of u that "depends on x only" in the more general sense proposed by the engineer. (Rather than show how this can be done, we prefer to let the mathematical readers figure it out by themselves, so they can see if they find it easy or hard, and develop some appreciation for this kind of question. The following hint will help. What has to be done is to start with a formula for u as a function of c, k, x and $\dot{x}$ that would result in the desired behavior. For example, we could take $u = (c-1)\dot{x} + (k-1)x$, which produces the equation $\ddot{x} + \dot{x} + x = 0$, all whose solutions obviously go to zero. Then, instead of just taking u to be a function of x and the true values of c, k, and $\dot{x}$, which are not known to us, we take u to be a function of x, y, $\hat{c}$ and $\hat{k}$, where y, $\hat{c}$ and $\hat{k}$ are estimated values of $\dot{x}$, $\hat{c}$ and $\hat{k}$. This should be supplemented with a set of equations governing the time evolution of y, $\hat{c}$ and $\hat{k}$, chosen so that the estimated values converge to the true values.)

The point of the above example is twofold: first, to show that even simple-looking control problems can be interesting, and second, to show that the mathematical questions that control engineers ask are often not exactly of the "prove or disprove" kind —with a precise mathematical formulation already given to them— that mathematicians normally expect to be asked. From these two facts it follows that there is a need for good books that explain the questions and results of control theory in a way that mathematicians can understand while doing justice to the engineering side and giving many examples of real applications.

The Control Handbook is an ambitious and spectacularly successful effort to respond to this need. It is a most welcome addition to the literature in the field, which should be especially interesting to mathematicians who want to understand control theory in its own terms, and be shown a broad perspective of the mathematical content of the theory and of its numerous applications. In an ideal world, this should mean most, or perhaps even all mathematicians.

Its monumental size should not deter anyone from taking a look. The book has 1548 very large two-column pages, and contains 121 articles. So this is a not a text one would read from cover to cover. But for those who are curious and willing to browse and explore, there is plenty here to satisfy all tastes. Most people who are "specialists in control theory" are really specialists in one rather narrow subfield, and the articles in other subfields will certainly contain new insights for them. As a reviewer, I have to confess that I have not read the whole book, cover to cover, but I have browsed and read several articles, both in areas close to my own interests, and on subjects far removed from them. The articles are carefully written and make enjoyable reading, a rather remarkable and unusual fact for collective works of this kind.

The book is divided into three parts devoted, respectively, to the fundamentals of control, advanced methods, and applications.

Part A contains articles on mathematical foundations (ordinary differential and difference equations, Fourier and Laplace transforms, linear algebra, and complex variables). This is followed by articles on models for dynamical systems (input-output, state space, graphical models, determining models), articles on continuous-time systems (stability tests such as Routh-Hurwitz and Nyquist, Nyquist and Bode plots, root locus, PID control, pole placement), digital control, analysis and design for nonlinear systems, and software.

Part B deals with more advanced methods of control, and contains articles on MIMO (multi-input multi-output) linear systems (both time-varying and time-invariant), Kalman filter and observers, linear quadratic control, $\mathcal{H}_\infty$ control, adaptive control, nonlinear control, system identification (a detailed article by L. Ljung), stochastic control, and control of distributed parameter systems. To give an idea of the thoroughness of the coverage of the various topics, let me use my own area, nonlinear control, as an example: the book has chapters by V. Jurdjevic on Lie brackets, by H. Khalil on two-time-scale and averaging and on Lyapunov stability, by F. Lamnabhi-Lagarrigue on Volterra and Fliess series, by A. Teel, T. Georgiou, L. Praly and E. Sontag on input-output stability, by A. Isidori on feedback linearization, by Isidori and C. Byrnes on zero dynamics, by Isidori on nonlinear output regulation, by R. Freeman and P. Kokotovic on Lyapunov design, by R. DeCarlo, S. Zak and S. Drakunov on sliding modes, by E. Abed, H. Wang and A. Tesi on control of bifurcations and chaos, by J. Baillieul and B. Lehman on oscillatory inputs, by M. Krstic and Kokotovic on adaptive nonlinear control, by K. Passino on intelligent control, by Passino and S. Yurkovich on fuzzy control, and by J. Farrel on neural control.

Part C, on applications, is the most removed from my own research work, and it is perhaps for that reason that I found it most enjoyable. One would hardly expect toilet tanks to be an exciting reading topic, but the article by B. Coury on "water level controls for the toilet tank: a historical perspective," turns out to be precisely that. For example, the author explains that "the Greeks and Romans had developed the types of technologies needed for the operation of modern toilets, but the widespread adoption of that technology did not occur for another 1800 years. One wonders, then, why a control technology so obviously useful took centuries to be adopted." His discussion of possible answers to this question is entertaining and informative. I know that from now on my courses on introduction to control theory for mathematicians are going to start with the toilet tank example.

The process control section has several other examples that will be of particular interest to those looking for "real life" applications: temperature control in large buildings, control of pH, control of pulp and paper making process, and an 11-author 22-page article on "control for advanced semiconductor device manufacturing: a case study." Then there are articles on automotive control systems, aerospace controls, control of robots and manipulators, power electronic control, control of power plants, and control of power transmission.

Every reviewer feels obliged to make at least one critical comment, so I shall conclude with my own: I was disappointed to see how little represented optimal control theory is in *The Control Handbook*. For a mathematician, optimal control is a venerable part of control theory, both because of its links with the calculus of variations and because of the tremendous amount of mathematical power needed to analyze optimal control problems. This is so to such an extent that many mathematicians automatically think of "control" as meaning "optimal control," and are not aware of the fact that optimal control is merely one of many areas within the vast field of control.

In view of the disproportionate importance attached to optimal control by mathematicians, a book like this one should be of great help to us, by giving us a healthier and more realistic perspective of how what matters to us is related to what the rest of the world truly cares for. Yet, I still believe there is more that is worth knowing about optimal control than one would gather from looking at the contents of this magnificent book. So, perhaps, my critical argument should be turned around and be read, constructively, as a call for a sequel —*The Control Handbook II?*— together with the wish that Part II be planned and edited with the same care, good taste, attention to detail, and breadth of scientific vision, as Part I was.

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