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Adaptive Stabilization and Disturbance Rejection for First-Order Systems[1]

by

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Abstract

We consider the use of a discontinuous adaptive control algorithm to achieve simultaneous stabilization and disturbance rejection for a first-order linear system. Lyapunov methods are used to prove stability of the closed-loop system and convergence to zero of the measurement. Closed-loop performance is compared to standard high-gain methods.

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1 Introduction

In this paper we consider the problem of feedback stabilization of a first-order linear system in the presence of exogenous disturbances. These disturbances include both plant disturbances and measurement noise. This case is of particular interest as it has been shown in [1] that several direct adaptive control schemes for minimum phase plants with relative degree 1 (such as those in [2, 3]), exhibit parameter divergence eventually leading to instability. In general, non-identifier based adaptive controllers based on high gain feedback tend to suffer from lack of robustness with respect to bounded disturbances.

A minimum-phase system with relative degree 1 can be stabilized by output feedback $u = -ky$, provided that $k > k_{\mathrm{s}}$, with k_{s} sufficiently large. To ensure that k grows beyond k_{s} the controller takes the form [2, 3, 4]

$$u = -ky, \tag{1}$$
$$\dot{k} = \gamma y^2. \tag{2}$$

With $y \to 0$ as $t \to \infty$ it can be proved [2, 4] that k converges. However, there is a practical limitation in implementing this controller. In the presence of sustained plant disturbance or measurement noise, k diverges eventually leading to instability. There have been several fixes to amend this problem in the literature and some of them are summarized below.

To avoid divergence of the gain k, (2) is replaced by [5]

$$\dot{k} = -\sigma k + \gamma y^2, \tag{3}$$

where $\sigma > 0$ is a small damping term. This modification yields bounded k for bounded y. However, (3) leads to an undesirable response [6] for unstable systems. To see this, suppose $|k|$ grows so that it is large enough to stabilize the unstable plant. As $y \to 0$, the term $-\sigma k$ dominates y^2 leading to a decrease in k, which destabilizes the system. This leads to bursting of the plant output. This behavior occurs regardless of the presence of disturbances.

To avoid bursting, [7] proposes the gain update law

$$\dot{k} = -\beta |y| k + \gamma y^2, \tag{4}$$

where σ in (3) is replaced by $\beta|y|$, the rationale being that such a term tends to zero with the measured output. However,(4) introduces new equilibria (p. 312 in [4]) which are functions of the tuning parameter β. The analysis of the resultant adaptive system is considerably difficult.

Another proposed modification to (2) is the *dead-zone modification* [4, 8, 9] given by

$$\dot{k} = \begin{cases} \gamma y^2, & |y| \geq \gamma\alpha, \\ 0, & |y| < \gamma\alpha, \end{cases} \tag{5}$$

where $\gamma > 0$ and α is a known bound on the disturbance or measurement noise. This scheme stops the gain adaptation when y is in a neighborhood of zero, assuring boundedness of all signals, but requiring knowledge of the disturbance bound α. Furthermore, since the modification contains a dead-zone, this method cannot assure asymptotic convergence of the plant output even when the disturbance is not present.

Adaptive stabilization algorithms that are robust to disturbances have been developed in the nonlinear context [10, 11, 12]. These results use various techniques that are not closely related to the algorithm presented in this paper.

In this paper, we provide an adaptive controller that guarantees convergence of the measured output in the presence of measurement noise as well as plant disturbances. In Theorem 2.1 we provide a controller for first-order relative degree 1 systems, which guarantees convergence of the measured plant output and boundedness of all parameters for unknown bounded measurement noise $d_{\rm m}$ and plant disturbance $d_{\rm p}$. As in [4, 13, 14] we assume the plant is minimum phase and that the order of the system, relative degree, and sign of the high frequency gain are known. For this case, the certainty equivalence controller is comprised of two elements, namely, a parameter estimator, and a disturbance rejection controller. Our proof of convergence is Lyapunov based.

2 Adaptive Disturbance Rejection

Consider the first-order system

$$\dot{q} + aq = bu + d_{\rm p}. \tag{6}$$

We assume that only the measurement

$$y = q + d_{\rm m} \tag{7}$$

is available for feedback and we assume that

(A1) a is constant but otherwise unknown;

(A2) b is constant, nonzero, and $\mu \triangleq \operatorname{sign}(b)$ is known, but b is otherwise unknown;

(A3) The plant disturbance $d_{\rm p}$ is bounded but otherwise unknown;

(A4) The measurement noise $d_{\rm m}$ is bounded and differentiable with bounded derivative but otherwise unknown.

The disturbance rejection problem constitutes finding a control input u such that $y(t) \to 0$ and k converges as $t \to \infty$.

Theorem 2.1 Consider the 2nd-order dynamic compensator

$$\begin{aligned} \dot{k} &= \gamma y^2, && (8)\\ \dot{\hat{\alpha}} &= \lambda \hat{\alpha}^{3/2}|y|, && (9) \end{aligned}$$

where γ and λ are positive constants, with the control input u given by

$$u = -\mu(ky + \hat{\alpha}\operatorname{sign}(y)). \tag{10}$$

Then $y \to 0$ and k converges as $t \to \infty$. Furthermore, $\hat{\alpha}$ is bounded, and $\inf_{t\geq 0} \hat{\alpha}(t) > 0$.

Proof. First note that the closed-loop system (6)-(10) is a differential equation with right-hand side discontinuity at $y = 0$. The closed-loop system can be regularized into a differential inclusion with $\mathrm{sign}(y_i)$ replaced by a set-valued map $\mathcal{S}(y_i)$, where $\mathcal{S}(y_i) = 1$ if $y_i > 0$, $\mathcal{S}(y_i) = -1$ if $y_i < 0$, and $\mathcal{S}(y_i) \in [-1, 1]$ if $y_i = 0$. The sufficient conditions given in [15] for the existence and continuity of solutions are satisfied, and the closed-loop differential inclusion has an absolutely continuous solution. Noting continuous dependence on the initial conditions and using Theorem 2.2 in [16] we conclude that the solution set of the closed-loop differential inclusion equals the solution set of the closed-loop system (6)-(10).

Next, using (6) and (7) we obtain

$$\dot{y} = -ay + bu + \psi, \tag{11}$$

where $\psi(t) \triangleq ad_{\mathrm{m}}(t) + \dot{d}_{\mathrm{m}}(t) + d_{\mathrm{p}}(t)$. Since $d_{\mathrm{m}}(t)$, $d_{\mathrm{p}}(t)$ and $\dot{d}_{\mathrm{m}}(t)$ are bounded, it follows that there exists $\alpha > 0$ such that

$$|\psi(t)| \leq |ad_{\mathrm{m}}| + |\dot{d}_{\mathrm{m}}(t)| + |d_{\mathrm{p}}(t)| \leq |b|\alpha, \quad t \geq 0. \tag{12}$$

Choosing $f > 0$, (10) and (11) yield

$$\dot{y} + fy = -|b|\tilde{k}y + \psi - |b|\hat{\alpha}\,\mathrm{sign}(y), \tag{13}$$

where $\tilde{k} \triangleq k - \frac{f-a}{|b|}$.

Next, let $\tilde{x} \triangleq \left[y, \tilde{k}, \hat{\alpha}\right]^{\mathrm{T}}$. Note that since f, a, and b are constant, $\dot{\tilde{k}} = \dot{k}$. Therefore the equations defining the state $\tilde{x}$ are given by (11), (8), (9) in the order of the components of $\tilde{x}$.

Now define $V : \mathbb{R}^2 \times (0, \infty) \to \mathbb{R}$ by

$$V(\tilde{x}) \triangleq \tfrac{1}{2}y^2 + \tfrac{|b|}{2\gamma}\tilde{k}^2 + \tfrac{2|b|}{\lambda}\left(\sqrt{\hat{\alpha}} + \tfrac{\alpha}{\sqrt{\hat{\alpha}}}\right) - \tfrac{4|b|\sqrt{\alpha}}{\lambda}. \tag{14}$$

Note that V is unbounded as $\hat{\alpha} \to 0$ and radially unbounded as $y, \tilde{k}, \hat{\alpha} \to \infty$. Furthermore, V has its minimum at $\tilde{x}_0 = [0, 0, \alpha]^{\mathrm{T}}$ at which $V(\tilde{x}_0) = 0$.

Suppose that $\hat{\alpha}$ vanishes on $(0, \infty)$ and let $t_1 = \min\{t > 0 : \hat{\alpha}(t) = 0\}$. Using the parameter update laws (8)-(9), we obtain, for $t \in [0, t_1)$,

$$\begin{aligned} \dot{V}(\tilde{x}(t)) &= -\tfrac{1}{2}fy^2 + y\psi - \hat{\alpha}|by| + \tfrac{|b|}{\gamma}\tilde{k}(\dot{k} - \gamma y^2) + |b|\tfrac{\hat{\alpha}-\alpha}{\lambda\hat{\alpha}^{3/2}}\dot{\hat{\alpha}} \\ &\leq -\tfrac{1}{2}fy^2 + |b|\tfrac{\hat{\alpha}-\alpha}{\lambda\hat{\alpha}^{3/2}}(\dot{\hat{\alpha}} - \lambda|y|\hat{\alpha}^{3/2}) \\ &= -\tfrac{1}{2}fy^2. \end{aligned} \tag{15}$$

Here we used $\psi y \leq \alpha|by|$, which follows from (12).

Hence, $V(\tilde{x}(t))$ is bounded for $t \in [0, t_1)$, which implies that $\alpha/\sqrt{\hat{\alpha}(t)}$ remains bounded for all $t \in [0, t_1)$. Hence, there exists $\delta > 0$ such that $\hat{\alpha}(t) > \delta$ for all $t \in [0, t_1)$. Since $\hat{\alpha}(t)$ is continuous, this contradicts $\hat{\alpha}(t_1) = 0$. Therefore the inequality (15) is valid for all $t \in [0, \infty)$, and thus $V(\tilde{x}(t))$ is bounded for all $t \in [0, \infty)$, which implies $\inf_{t\geq 0} \hat{\alpha}(t) > 0$. It follows that $V(\tilde{x}(t))$ has a limit as $t \to \infty$. Hence all components of $\tilde{x}(t)$ are bounded for $t \in [0, \infty)$.

Next, note that, for $t \geq 0$,

$$\int_0^t \tfrac{1}{2} f y^2(\tau) d\tau \leq V(\tilde{x}(0)) - V(\tilde{x}(t)). \tag{16}$$

Since the right-hand-side of (16) is bounded as $t \to \infty$, it follows that $y \in \mathcal{L}_2$. Since $\tilde{x}$ is bounded it follows that u is bounded. From (12) it follows that ψ is bounded. Since y is bounded it follows from (11) that $\dot{y}$ is bounded. Therefore Barbalat's lemma [17] implies that $y \to 0$ as $t \to \infty$. From (8) note that

$$\lim_{t\to\infty}(k(t) - k(0)) = \int_0^\infty \dot{k}(\tau) d\tau = \gamma \int_0^\infty y^2(\tau) d\tau. \tag{17}$$

Since $y \in \mathcal{L}_2$ it follows that k converges as $t \to \infty$. $\square$

Remark 2.1 The variable $\hat{\alpha}$ given by (9) attempts to estimate the overall disturbance bound characterized by α in (12). The variable $\hat{\alpha}$ in conjunction with the switching term in (10) achieves disturbance rejection. However, if d_{m} and d_{p} are identically zero, then it suffices to omit (9) and set $\hat{\alpha} = 0$ in (10). The resulting simplified controller is equivalent to (1), (2) and can be shown to guarantee $y = q \to 0$ as $t \to \infty$. Therefore the controller presented in Theorem 2.1 achieves two purposes simultaneously, namely, adaptive stabilization and adaptive disturbance rejection.

3 Numerical Example

The control law (8)-(10) presented in Theorem 2.1 is discontinuous at $y = 0$, which makes exact implementation impractical. The signum function constitutes infinite gain control at $y = 0$, allowing for perfect disturbance rejection, but causing persistent control chatter once y reaches a neighborhood of 0. We therefore consider an approximation of the control law in Theorem 2.1 that does not have a discontinuity at $y = 0$. This approximate controller also includes a deadzone on the gain adaptation to prevent gain blow-up caused by imperfect disturbance cancellation.

Consider the plant (6), (7) with parameters $a = -4$ and $b = 0.5$ and noise signals

$$d_{\mathrm{p}} = \tfrac{1}{2}\sin(2\pi t) + \tfrac{1}{2}\cos(10t), \tag{18}$$
$$d_{\mathrm{m}} = \tfrac{1}{2}\sin(4et) + \tfrac{1}{2}\cos(7t). \tag{19}$$

Figure 1 shows the dual-tone disturbances d_{p} and d_{m} used in the examples.

Define $\delta_\varepsilon : \mathbb{R} \to \{0, 1\}$ and $\mathrm{sat}_\varepsilon : \mathbb{R} \to [-1, 1]$ given by

$$\delta_\varepsilon(x) = \begin{cases} 0, & |x| \leq \varepsilon, \\ 1, & |x| > \varepsilon, \end{cases} \tag{20}$$

$$\mathrm{sat}_\varepsilon(x) = \begin{cases} -1, & x < -\varepsilon, \\ x/\varepsilon, & |x| \leq \varepsilon, \\ 1, & x > \varepsilon. \end{cases} \tag{21}$$

Figure 2 illustrates δ_ε and sat_ε. Now replace the control equations (8)-(10) with

$$\begin{aligned}
\dot{k} &= \gamma y^2 \delta_{0.01}(y), & (22)\\
\dot{\hat{\alpha}} &= \lambda \hat{\alpha}^{3/2} |y| \delta_{0.01}(y), & (23)\\
u &= -\mu(ky + \hat{\alpha}\,\mathrm{sat}_{0.01}(y)). & (24)
\end{aligned}$$

Choose the controller parameters $\gamma = 1$ and $\lambda = 1$. Figures 3 and 4 show the response of the closed-loop system (6), (7), (22)-(24). Note that k and $\hat{\alpha}$ converge even with imperfect implementation of the sign function due to the use of a deadzone in (22) and (23). In Figure 3 the closed-loop response is compared to (6), (7), (1), (5). Controller (22)-(24) is shown to have significantly better disturbance rejection capability, yet uses similar control effort after the initial transient period.

Figure 5 shows y with an expanded vertical scale. Note that due to the use of δ_ε and sat_ε in (22)-(24), y does not converge asymptotically to zero but remains in a small neighborhood of zero.

4 Concluding Remarks

The controller presented in Theorem 2.1 is a high-gain adaptive-control algorithm that performs stabilization and disturbance rejection in the presence of bounded plant disturbances and measurement noise. The discontinuous functions present in the controller can be implemented imperfectly without loss of stability or significant degradation of performance. This algorithm represents an alternative to standard adaptive schemes, which may be unstable in the presence of disturbances (2). Other standard adaptive schemes maintain stability in the presence of disturbances, but are not formulated to reject the disturbances perfectly (3), (4), (5).

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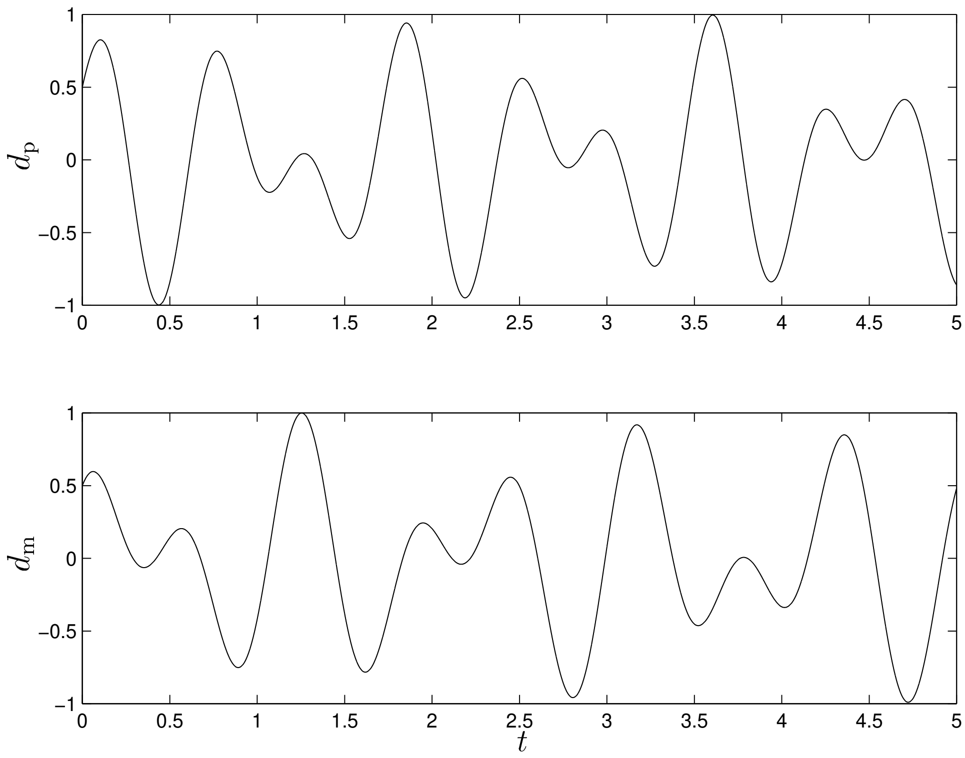

Figure 1: Noise signals d_{p} and d_{m}

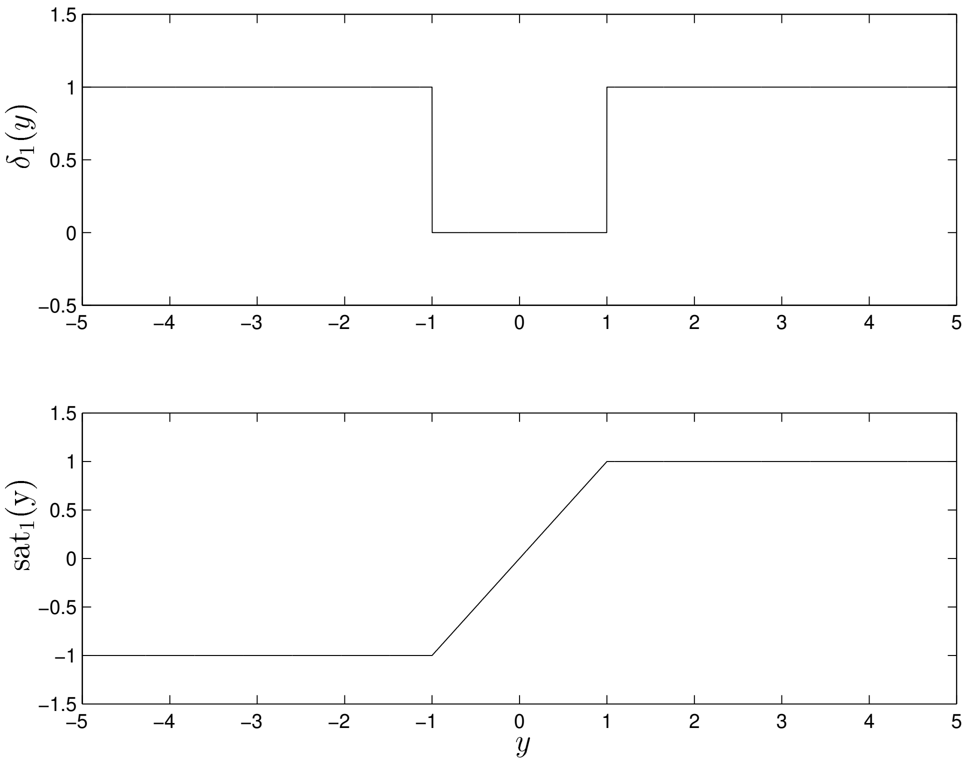

Figure 2: Functions δ_ε and sat_ε

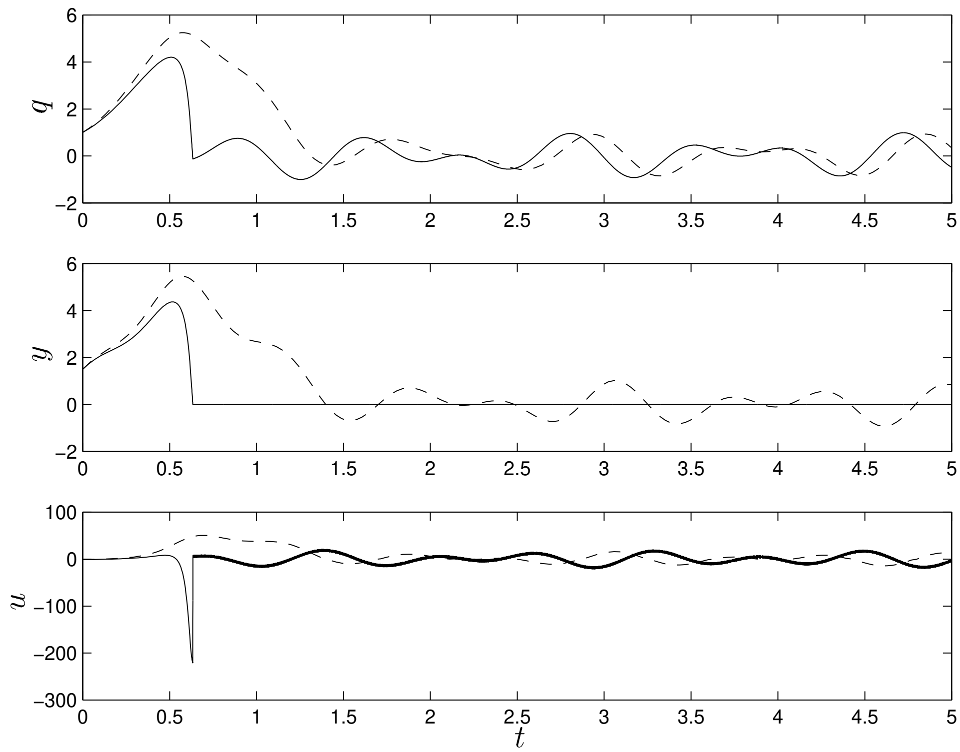

Figure 3: System response using controller (22)-(24), shown with solid line, compared to response using controller (1), (5), shown with dashed line

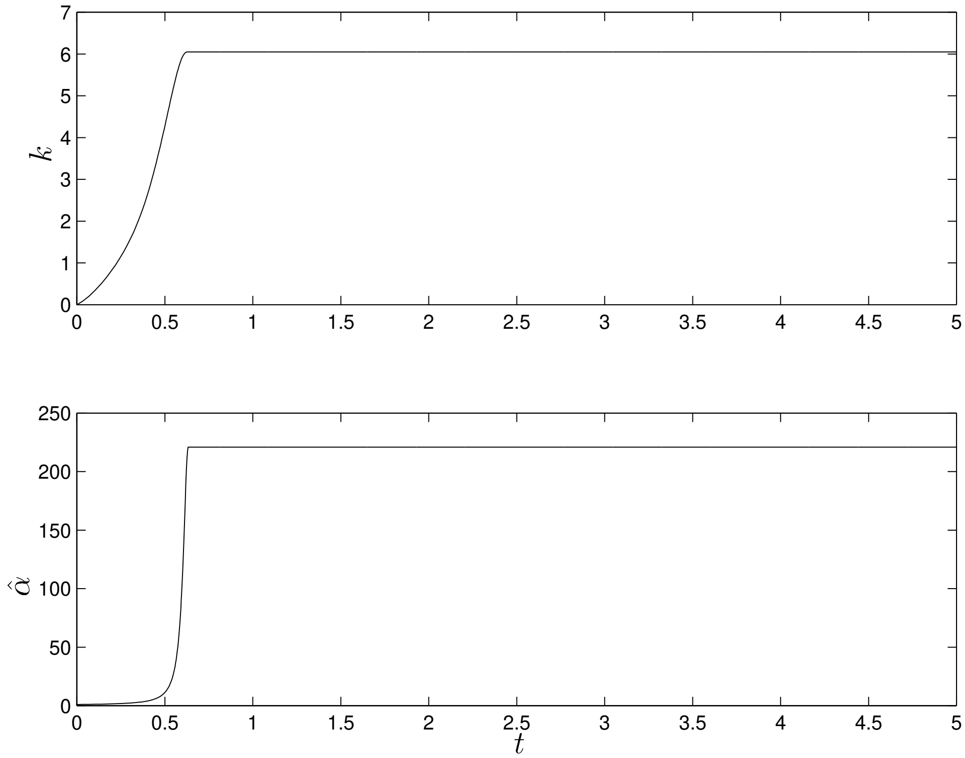

Figure 4: Time history of k and $\hat{\alpha}$.

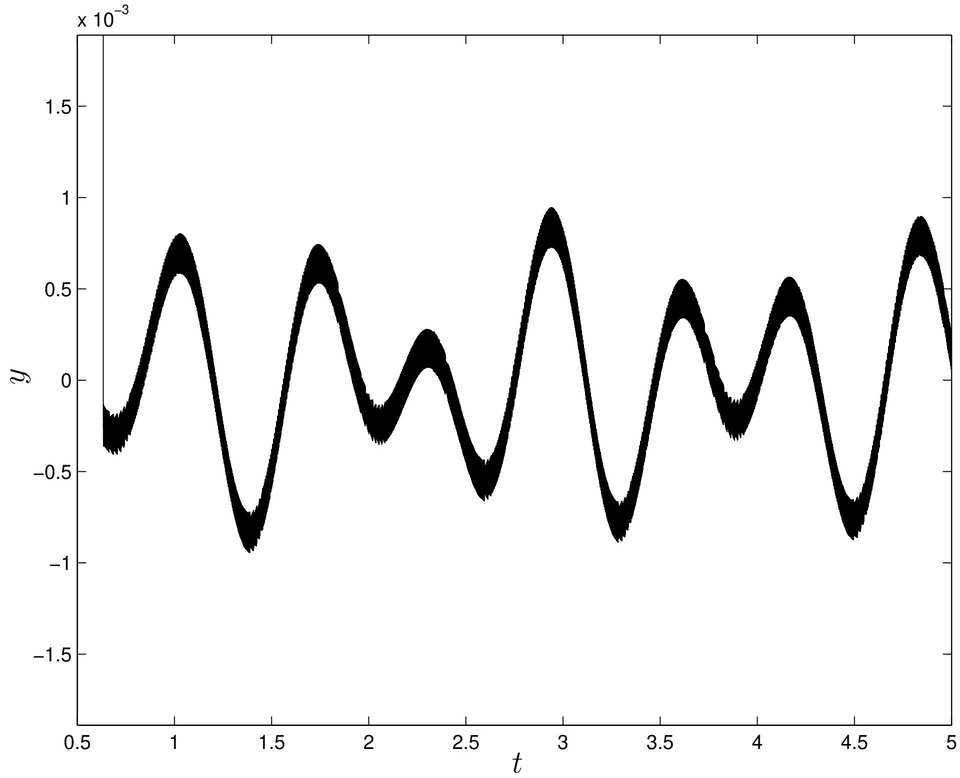

Figure 5: Time history of y with expanded vertical scale