350 SECOND MIDTERM FALL 2019

Question 1.

Find the general solution of the following system of linear equations:

$$x_1 - 2x_3 + x_4 = -1$$

$$2x_1 - x_2 + x_3 - 3x_4 = -9$$

$$9x_1 - 3x_2 - x_3 - 8x_4 = -32$$

Question 2.

(i) Compute the inverse of the following matrix:

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 0 & -1 \\ 1 & 0 & -1 \end{pmatrix}$$

(ii) Solve the following system of linear equations:

$$x_1 - x_2 = 1$$

$$2x_1 - x_3 = 3$$

$$x_1 - x_3 = 1$$

Question 3.

Let $A \in \mathrm{M}_{3\times 3}(\mathbb{R})$ be the matrix

$$A = \begin{pmatrix} 1 & -3 & 3 \\ 0 & -5 & 6 \\ 0 & -3 & 4 \end{pmatrix}$$

Determine whether A is diagonalizable; and if so, find an invertible matrix Q such that $Q^{-1}AQ$ is a diagonal matrix.

Question 4. Let A be an $n \times n$ matrix over a field F.

(i) Give the definition of an eigenvector and eigenvalue of A.

(ii) Prove that if λ is an eigenvalue of A and $m \geq 1$ is a positive integer, then λ^m is an eigenvalue of A^m.

The $n \times n$ matrix A is said to be nilpotent if there exists a positive integer $\ell \geq 1$ such that $A^\ell = \mathbf{0}$ is the zero matrix.

(iii) Prove that if A is nilpotent and λ is an eigenvalue of A, then $\lambda = 0$.

(iv) Prove that if A is nilpotent, then 0 is an eigenvalue of A.