350 SECOND MIDTERM SPRING 2020

Question 1.

Find the general solution of the following system of linear equations:

$$2x_1 - 8x_2 + x_3 - 4x_4 = 9$$
$$x_1 - 4x_2 - x_3 + x_4 = 3$$
$$3x_1 - 12x_2 - 3x_4 = 12$$

Question 2.

(i) Compute the inverse of the following matrix:

$$A = \begin{pmatrix} 2 & -3 & 4 \\ 3 & -6 & 8 \\ 1 & -1 & 1 \end{pmatrix}$$

(ii) Solve the following system of linear equations:

$$2x_1 - 3x_2 + 4x_3 = 5$$
$$3x_1 - 6x_2 + 8x_3 = 8$$
$$x_1 - x_2 + x_3 = 2$$

Question 3.

Let $A \in \mathrm{M}_{3\times 3}(\mathbb{R})$ be the matrix

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

Determine whether A is diagonalizable; and if so, find a diagonal matrix D and an invertible matrix Q such that $Q^{-1}AQ = D$.

Question 4. Throughout this question, let $A \in M_{n\times n}(F)$ be an $n \times n$ matrix over a field F.

(i) Give the definition of an eigenvector and an eigenvalue of A.

(ii) Suppose that λ is an eigenvalue of A. Give the definition of the corresponding eigenspace E_λ.

From now on, let $B \in M_{n\times n}(F)$ be an $n \times n$ matrix over F such that $AB = BA$. For each eigenvalue λ of A, let E_λ be the corresponding eigenspace.

(iii) Prove that if λ is an eigenvalue of A and $v \in E_\lambda$, then $Bv \in E_\lambda$.

(iv) Prove that if A has n distinct eigenvalues, then B is diagonalizable.

(*Remark: This is not a typo. I really do want you to prove that B is diagonalizable.*)

Of course, in class, we have seen that if A has n distinct eigenvalues, then A is diagonalizable. The following shows that (iv) is not true if we only assume that A is diagonalizable.

(v) Give examples of 2×2 matrices D, $B \in M_{2\times 2}(\mathbb{R})$ such that D is diagonal and $DB = BD$, but B is *not* diagonalizable.