Turn in starred problems (including problem 3.A) on Wednesday 09/28/2016. Do not turn in solutions for any unstarred problems.

Multiple-page homework must be STAPLED when handed in.

Section 4.3:

- 6 *(b), (f), (v)

Section 4.5

- 8, *9

Section 4.6

- *2, *5(a) (**see instructions in Remark 3 below**), 6, 16

*3.A (a) The point $x = 1$ is a regular singular point for the Legendre equation $(1-x^2)y'' - 2xy' + \lambda y = 0$, and the indicial equation is $r^2 = 0$. Verify these things.

(b) From (a) we know that there will be solutions $\tilde{y}_1(x) = \sum_{n=0}^{\infty} a_n(x-1)^n$ and $\tilde{y}_2(x) = \tilde{y}_1(x)\ln|x-1| + \sum_{n=1}^{\infty} b_n(x-1)^n$. Find $\tilde{y}_1(x)$ and show that if $\lambda = n(n+1)$ for some nonnegative integer n then $\tilde{y}_1(x)$ is a polynomial, and give this polynomial for $n = 0, 1, 2$ and 3. **Verify that your results agree with Table 1 on page 213.** (You are *not* asked to find $\tilde{y}_2(x)$).

Remarks: 1. The problems from 4.2.6 illustrate the various possibilities for the Frobenius method. In Assignment 2 we solved 4.3.6(t), where $r_1 - r_2$ is not an integer; all the remaining cases of Theorem 4.3.1 occur in the problems above, and for case (iii) there is an example in which there is a logarithm in the solution and one in which there is not. I am asking you to hand in only one of these, but I strongly suggest that you work out all of them at least to the point that you can see the second solution emerging.

2. Problem 4.5.9 has two parts: you are first asked to express $\Gamma(1/2)$ in terms of the *Gaussian integral* $\int_0^\infty e^{-u^2}\,du$, then to evaluate this integral. Gaussian integrals like this are important in various applications, and it is nice to know the standard trick (explained in the problem) for evaluating them. For example, the normal distribution of a random variable is important in all statistical considerations, and to work with these distributions you need to know the value of a Gaussian integral.

3. The idea in problems 4.6.2 and 4.6.5(a) is to derive equations (14a,b) in two ways. In 5(a), as instructed, you are to start with the series (9) and (10) and plug in $\nu = 1/2$. In 2, solve the Bessel equation with $\nu = \pm 1/2$ from the beginning of the Frobenius method. In the process, verify that there is no logarithm in the second solution.

4. I don't expect you to work out 4.6.16 in detail, but take a look. This is another example of a concrete physical problem needing Bessel functions for its solution.