

Warning: not all of these answers have been carefully checked. If you disagree with one of them, you may be correct

1. (a)  $r_1/r; r_2/r; \binom{r_1}{2}/\binom{r}{2}, r_1r_2/\binom{r}{2}, \binom{r_2}{2}/\binom{r}{2}$ .

(b)  $(r_1 + 3r_2)/r, 4r_1r_2/r^2, -4r_1r_2/r^2(r - 1)$ .

(c)  $n(r_1 + 3r_2)/r, 4n(n - 1)r_1r_2/r^2(r - 1)$ .

2. (a) Gamma distribution,  $\lambda = 3/2, \alpha = 2$ . (b)  $c = 1$ , (c)  $3/2, 2 \log 2, 2 - 3 \log 2$ .

(d) Let  $u = \sqrt{2z}$ ,  $\phi(v) = e^{-v^2/2}/\sqrt{2\pi}$ ,  $\Phi(v) = \int_{-\infty}^v \phi(u)du$ . Then  $f_Z(z) = 3(\Phi(2u) - \Phi(u))/2u^3 + (4u^3 - 3)\phi(2u)/u^2 - (u^2 - 3)\phi(u)/2u^2$  (hard!).

3. (a)  $1/4$ . (b)  $1 - \Phi((35 - 25)/(30\sqrt{3}/4)) = 1 - \Phi(0.77) = 0.2206$ .

4. (a)  $\binom{6}{5}p^5(1 - p) + \binom{6}{6}p^6$ . (b)  $\sum_{k=0}^3 \left[ \binom{3}{k}p^k(1 - p)^{3-k} \right]^2$ . (c)  $p(1 - p)^3$ .

5. (a)  $P(\text{Urn } k) = 6/11, 3/11, 2/11$  for  $k = 1, 2, 3$ ;

(b)  $P(\text{second ball is number } k) = 49/66, 13/66, 4/66$  for  $k = 1, 2, 3$ .

6.  $f_Z(z) = 0, z < 0$ ;  $f_Z(z) = z/2 - 1/6 - e^{-3z}/6, 0 \leq z < 2$ ;  $f_Z(z) = 7e^{-3(z-2)}/6 - e^{-3z}/6, 2 \leq z$ .

7. (a)  $E[X] = 25, \text{Var}(X) = 25/3$ . (b)  $\Phi(.69) - \Phi(-1.39) = .6726$ . (c)  $a = 9.13$ .

8. (a)

$y \setminus x$	1	2	3	4	5	6	$p_Y(y)$
0	5/36	4/36	3/36	2/36	1/36	0	5/12
1	1/36	2/36	3/36	4/36	5/36	6/36	7/12
$p_X(x)$	1/6	1/6	1/6	1/6	1/6	1/6	

(b)  $7/2, 7/12, 35/12, 35/144, 35/72$ .

(c)

$z$	1	2	3	4	5	6	7
$p_Z(z)$	5/36	5/36	5/36	5/36	5/36	5/36	6/36

9. (i)  $13 \cdot \binom{12}{2} \cdot 4 \cdot 4 / \binom{52}{6}$ , (ii)  $\binom{13}{3} \binom{4}{2}^3 / \binom{52}{6}$ , (iii)  $\binom{13}{2} \binom{4}{3}^2 / \binom{52}{6}$ , (iv)  $\left[ \binom{13}{2} \binom{4}{3}^2 + 13 \cdot 12 \cdot \binom{4}{4} \binom{4}{2} \right] / \binom{52}{6}$ , (v)  $8 \cdot 4^6 / \binom{52}{6}$ , (vi)  $\binom{13}{6} 4^6 / \binom{52}{6}$ , (vii)  $13 \cdot 12 \cdot 11 \cdot \binom{4}{3} \binom{4}{2} \binom{4}{1} / \binom{52}{6}$ .

10.  $M_X(t) = e^t(e^{nt} - 1)/n(e^t - 1)$ ;  $E[X] = (n + 1)/2$ ;  $\text{Var}(X) = (n^2 - 1)/12$ . Use formulas  $\sum_{k=1}^n k = n(n + 1)/2$ ,  $\sum_{k=1}^n k^2 = n(n + 1)(2n + 1)/6$ .

11. See the book.

12.  $10/73$ .

13. (a)  $E[X_i] = 1 - \binom{19N}{10} / \binom{20N}{10}$ ,  $\text{Var}(X_i) = \left( 1 - \binom{19N}{10} / \binom{20N}{10} \right) \left( \binom{19N}{10} / \binom{20N}{10} \right)$ ,

$\text{Cov}(X_i, X_j) = E[X_i X_j] - E[X_i]E[X_j] = 1 - 2 \binom{19N}{10} / \binom{20N}{10} + \binom{18N}{10} / \binom{20N}{10} - \left( 1 - \binom{19N}{10} / \binom{20N}{10} \right)^2$ ,  
 $E[X] = 20E[X_i]$ ,  $\text{Var}(X) = 20 \text{Var}(X_i) + (20 \cdot 19) \text{Cov}(X_i, X_j)$ .

(b)  $X = 10, E[X] = 10, \text{Var}(X) = 0$ . Check:  $E[X_i] = 1/2, \text{Var}(X_i) = 1/4, \text{Cov}(X_i, X_j) = -1/76$ .

(c)  $E[X_i] = 1 - (19/20)^{10}$ ,  $\text{Var}(X_i) = (1 - (19/20)^{10})(19/20)^{10}$ ,

$\text{Cov}(X_i, X_j) = 1 - 2(19/20)^{10} + (18/20)^{10} - (1 - (19/20)^{10})^2$ ,

$E[X] = 20E[X_i]$ ,  $\text{Var}(X) = 20 \text{Var}(X_i) + (20 \cdot 19) \text{Cov}(X_i, X_j)$ .

(d)  $\binom{19N}{10} / \binom{20N}{10} = \binom{19N}{20N} \binom{19N-1}{20N-1} \cdots \binom{19N-9}{20N-9} \rightarrow \left( \frac{19}{20} \right)^{10}$ , etc.

14.  $(3^{n-1} - 2^n + 1)/3^{n-1}$ .

15. (a)  $4/13$ ; (b)  $88/169$ .

16. See book or class notes.