Assignment 1

Turn in starred problems Wednesday, January 25, at the beginning of the period. See the remarks below for hints or modifications of several of these problems.

Exercises from Abbott, Understanding Analysis:

Section 1.2: 3, 4, 6(d)*, 11* Section 1.3: 6, 7, 8, 10*, 11 Section 1.4: 1, 2, 5, 8*

1.A* Suppose that A and B are nonempty sets of nonnegative real numbers (that is, $A, B \subset \mathbb{R}_+$, where $\mathbb{R}_+ = \{x \in \mathbb{R} \mid x \ge 0\}$) and that A and B are bounded above. Let $AB = \{ab \mid a \in A, b \in B\}$. Prove that $\sup AB = (\sup A)(\sup B)$.

Remarks, hints, and further instructions:

2.3 As well as giving a counterexample for the false statements, give a proof for the statements which are true.

3.6(d) A very useful corollary of the triangle inequality.

3.10 Part (b) here is not very clearly stated; \mathbb{R} *does* satisfy the Axiom of Completeness (that is part of its definition), so in that sense there is nothing to prove. A better formulation: (b) Suppose that \mathbb{F} is an ordered field which satisfies the Cut Property. Prove that \mathbb{F} satisfies the Axiom of Completeness.

4.8 Let us take it that by "provide a compelling argument", Abbott means "provide a proof."

1.A Compare with Exercise 1.3.6.