**Workshop 13**
**November 30, 2001**

*Graphing Curves in Parametric Form on the TI-82:* To plot the curve with parametric equations $x = f(t)$ and $y = g(t)$, press the MODE key, move the cursor down to fourth line and over to `Par`, and press ENTER. Now press $Y =$ key and insert the parametric formulas $X_{1T} = f(T)$ and $Y_{1T} = g(T)$. Then press the WINDOW key and set the range for the parameter $T$ and the $T_{\text{step}}$ size. (A typical step size might be .1, but you should experiment to see what happens when you change this. Some experimentation with the range of $T$ may also be useful: with too small a range, you don't see all the curve, but too large a range wastes time, as the calculator tries to graph points outside the $x$-$Y$ window.) You must also set the $X$-$Y$ window size as usual. Then press GRAPH. The TRACE command is very useful here, as it displays the $T$ values as well as the $X$-$Y$ values. *Similar procedures apply to other graphing calculators.*

**1.** In this problem we wish to approximate $\ln x$ for $1 \le x \le 2$.

(a) Find the Taylor polynomials $T_0(x)$, $T_1(x)$, $T_2(x)$, and $T_3(x)$, with center $a = 1$, approximating $\ln x$ near 1. Graph these polynomials, and $\ln x$, for $1 \le x \le 2$. Does the polynomial $T_n$ approximate $\ln x$ better as $n$ gets larger? Use your calculator to compute the actual error at $x = 2$ in each of these approximations.

(b) Use Taylor's formula $R_n(x) = \dfrac{f^{(n+1)}(z)}{(n+1)!}(x-a)^{n+1}$ (where $a < z < x$ when $a < x$) to find the maximum possible error in the approximation $\ln x \simeq T_3(x)$; your answer should be a single number valid for all $x$, $1 \le x \le 2$. How close is this to the actual error when $x = 2$?

(c) Explain why Taylor's inequality (as given on page 754 of Stewart) can not be used to estimate the error for all $x$ satisfying $1 \le x \le 2$. Hint: what $M$ would satisfy $|f^{(4)}(x)| \le M$ for $|x - 1| \le 1$?

**2.** A particle $P$ travels on the parabola $x = y^2$. Give two possible parametric formulas for the position of $P$, obtained as follows. (The parametric formulas may have to be given "piecewise".)

(a) Use $\theta$ as parameter, where $\theta$ is the angle that the ray from the origin to $P$ makes with the positive $x$-axis. That is, describe the parabola by two functions $x(\theta)$ and $y(\theta)$, with $\theta$ as above. Draw a picture to help you see how to proceed!

(b) Use $r$ as parameter, where $r$ is the distance from the origin to $P$. Draw a picture.

**3.** Use your calculator to sketch each of the following parametric curves, for $0 \le t \le 2\pi$. Then identify each curve precisely by using the algebraic properties of $\sin t$ and $\cos t$ to find an equation of the curve in terms of $x$ and $y$ coordinates. (The identity $\sin^2 t + \cos^2 t = 1$ is sometimes useful.)

(a) $x(t) = \sin t + \cos t$ (Hint: compute $x^2$.)
$y(t) = \sin 2t$

(b) $x(t) = 2\cos t$
$y(t) = \sin t$

(c) $x(t) = \sin t + 2\cos t$
$y(t) = 2\sin t - \cos t$

(d) $x(t) = \sin t$
$y(t) = \csc t$

**4.** Compute

$$\lim_{x \to 0} \frac{(\sin 3x - 3x)^2}{(e^{2x} - 1 - 2x)^3}.$$

Use Maclaurin series for $\sin 3x$ and $e^{2x}$ (easily obtained from series you already know) instead of L'Hôpital's Rule.