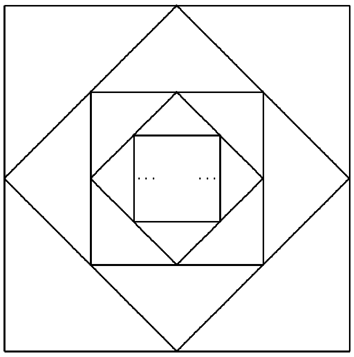
Workshop 8
October 26, 2001



1. Suppose that you draw a $2'' \times 2''$ square, then join the midpoints of its sides to make another square, then join the midpoints of *that* square's sides to form another square, etc., and that in fact you draw an infinite sequence of squares in this way. What is the sum of the perimeters of *all* the squares?

2. Two of the following statements are (always) true, and two are (sometimes) false. Decide which are which. Then for each true statement, explain briefly why it must be true; for each false statement, give an example of a series for which it is not true.

(a) If a series diverges, then its n-th term does not approach 0 as $n \to \infty$.

(b) If the n-th term of a series does not approach 0 as $n \to \infty$, then the series diverges.

(c) If a series converges, then its partial sums form a bounded sequence.

(d) If the partial sums of a series form a bounded sequence, then the series converges.

3. (a) Using the integral test, show that the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ converges.

(b) Let s denote the sum of this series: $s = \displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$. If you attempted to estimate s by computing a partial sum of the series—that is, by adding up the first few terms—how many terms would you have to include to guarantee that the error (the difference between s and the sum you calculate) would be less than 0.05? Explain why you are *certain* of this accuracy. Use the inequality

$$\sum_{n=N+1}^{\infty} f(n) \leq \int_N^{\infty} f(x)\,dx. \tag{$*$}$$

You may ignore any calculator (round-off) error arising in your addition.

Remark; The inequality $(*)$ is based on the ideas of the integral test and is explained by Figure 3 on page 719 of the text.

4. Consider the sequences $\quad a_n = \left(1 + \dfrac{1}{n}\right)^n, \quad b_n = \left(1 + \dfrac{1}{n^2}\right)^n, \quad$ and $\quad c_n = \left(1 + \dfrac{1}{\sqrt{n}}\right)^n.$

(a) Using your calculator (try the `TABLE` feature; `TblSet` is useful, too), find and plot the first ten terms of each sequence. Using this information, try to guess the limit of each sequence.

(b) Use L'Hôpital's Rule to find the limit of each sequences. (Hint: take logarithms, then replace n by x and let x tend to infinity, or replace n by $1/x$ and let x tend to zero.)