Workshop 7
October 19, 2001

1. Four colonies of bacteria have the following population growth patterns:

Colony II doubles its population every two days;

Colony III triples its population every three days;

Colony IV quadruples its population every four days;

Colony V quintuples its population every five days.

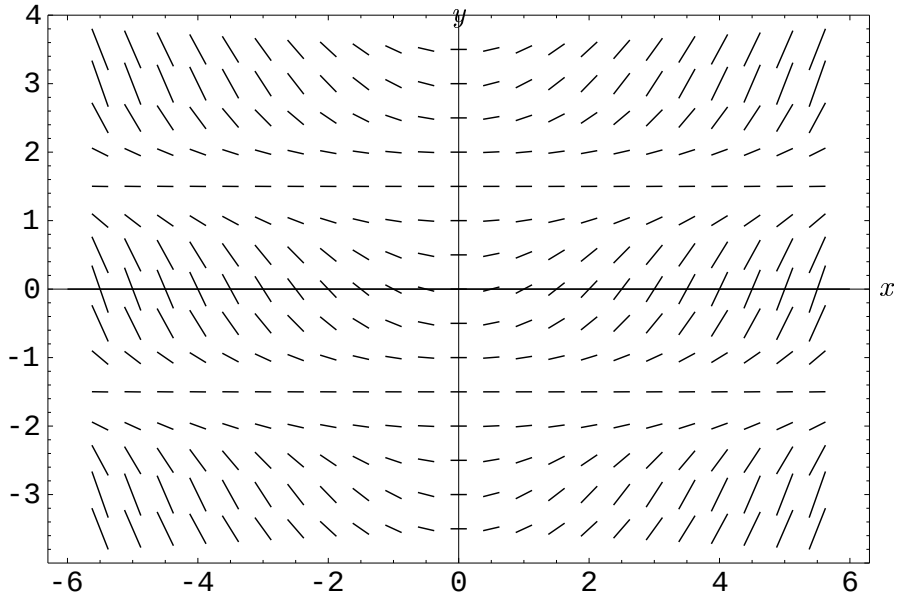
(a) Find the growth constant (the constant k in the equation $\dfrac{dP}{dt} = kP$) for each colony.

(b) Which colony is growing most rapidly? Which colony is growing most slowly? Which colonies, if any, are growing at the same rate?

2. Consider the differential equation $\dfrac{dy}{dx} = x\cos^2(y)/2$. Its direction field is plotted below.

(a) Using the direction field, sketch the solution curve that satisfies the initial condition $y(2) = 0$.

(b) What is dy/dx when $y = \pi/2$? How is this reflected in the direction field below? Sketch the solution of the differential equation satisfying the initial condition $y(0) = \pi/2$.



(c) Find explicitly the general solution of the equation and then the solution $y = f(x)$ that satisfies the initial condition $y(2) = 0$. Using your calculator, plot this function. Include a separate sketch of this plot with your workshop (do *not* plot it on the direction field).

(d) Can the solution sketched in (b) be obtained from the general solution by choice of constant?

3. Find the general solution of the differential equation $e^{-y}\dfrac{dy}{dt} + \cos t = 0$.

(a) Sketch the solutions corresponding to several values of the constant C of integration (use the graphing function of your calculator). Does every value C of the constant of integration correspond to a solution curve? If not, which values do?

(b) Do all the solutions have the same domain? Explain. Give an specific C for which all real numbers t belong to the domain.