Workshop 6
October 12, 2001

1. Assume that a is a positive constant. Let R be the region bounded above by $y = 1/x^a$, below by $y = 0$, and on the left by the line $x = 1$. Determine those values of a for which the integral that results from attempting to calculate each of the following converges:

(a) the volume of the solid obtained by rotating R around the x-axis;

(b) the volume of the solid obtained by rotating R around the y-axis.

(c) The lateral surface area of the solid described in (a).

2. Suppose that the functions f and g are defined and positive on some interval $[a, \infty)$.

- If $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$, we say that f *grows more slowly than* g or that g *grows more rapidly than* f, as $x \to \infty$. This may be indicated by writing $f \ll g$. (We use this terminology even if f and/or g don't in fact "grow" as $x \to \infty$; for example, they might go to zero).

(a) Arrange the following eight functions in order of rate of growth, from least rapidly growing to most rapidly growing. indicating any cases in which the functions grow at comparable rates. Justify your answers. It is helpful to first check that if $f \ll g$ and $g \ll h$ then $f \ll h$.

$$x; \quad \ln x; \quad \sqrt{x^3 + x}; \quad e^x; \quad \ln(\ln x); \quad \ln\left(e^{x^2} + 1\right); \quad x\ln(x); \quad e^{x^2}.$$

(b) Arrange the reciprocals of the same ten functions in order of rate of growth. (Hint: if f and g are related by $f \ll g$, what is the relation between $1/g$ and $1/f$?)

(c) For which of the reciprocals $h(x) = 1/f(x)$ in (b) does the integral $\int_{100}^{\infty} h(x)\,dx$ converge? Suggestion: Use the *Comparison Theorem* on page 530 of the text. To do so you will need to answer this queston: if $f \ll g$ is it true that $f(x) \le g(x)$? or true for some x?

Remark: If $\lim_{x\to\infty} \frac{f(x)}{g(x)} = L$, with L a **positive** constant (not 0), then we say that f and g *grow at comparable rates* as $x \to \infty$. This may be indicated by writing $f \sim g$; for example, $x^{3/2} \sim \sqrt{x^3 + x}$. This concept is often important but is not needed in the problem above.

3. (a) Use the formula for arc length to verify that the circumference of a circle of radius r is $2\pi r$.

(b) Use the formula for area of a surface of revolution to verify that the area of a sphere of radius r is $4\pi r^2$.