Workshop 3

September 21, 2001

1. Calculate four of the five indefinite integrals:

(a) $\int x\cos x^2\,dx$ (b) $\int x^2\cos x\,dx$ (c) $\int x^2\cos x^2\,dx$

(d) $\int x^2\cos x^3\,dx$ (e) $\int x^3\cos x^2\,dx$

2. A body of mass 2 kilogram moves along a line; its position relative to the origin at time t seconds is $s(t)=t^3$ meters.

(a) Using Newton's law ($F=ma$) find the force $F=f(t)$ which acts on the body at time t, and then find the average value of this force between $t=0$ sec and $t=2$ sec. This is the *time average* $F_{\text{time average}}$ of the force. Draw a graph of the function $f(t)$ for $0\le t\le 2$. Find the time t^* for which $f(t^*)=F_{\text{time average}}$ and give a graphical interpretation.

(b) Find a formula $F=g(s)$ for the force which acts on the body when it is at position s, and then find the averge value of $g(s)$ between $s=s(0)$ and $s=s(2)$. This is the *distance average* $F_{\text{distance average}}$ of the force as the body moves from position $s(0)$ to position $s(2)$. Draw a graph of the function $g(s)$ for $s(0)\le s\le s(2)$. Find the position s^* for which $g(s^*)=F_{\text{distance average}}$ and give a graphical interpretation.

Notice that the time and distance averages of the force are different!

(c) For extra credit (a bit of physics is required): Explain the formulas

$$\text{(change of momentum)} = \text{(time average of force)} \times \text{(elapsed time)}\ ;$$
$$\text{(work done)} = \text{(distance average of force)} \times \text{(displacement)}\ .$$

3. Suppose that $f(x)$ is defined and nonnegative for all $x\ge 0$ and that, for $t>0$, R_t is the region bounded by $y=0$, $y=f(x)$, $x=0$, and $x=t$. We are given that if R_t is rotated around the x-axis, the volume of the resulting solid is $\frac{\pi}{3}(2t^3+3t)$. If the same region is rotated around the y-axis, what is the volume of the solid obtained?

If you write up this problem, include a clear discussion and sketches (discs, washers, shells, …) showing how you calculated any volumes.

4. Consider the family of functions $f_n(x)=x\sin nx$, one function for each value of n, $n=1,2,\ldots$.

(a) On a single set of axes, sketch the graph of $f_4(x)$, together with the graphs of $y=x$ and $y=-x$, for $0\le x\le 2\pi$. Then do the same with f_8 in place of f_4. What would the picture look like when n is very very large?

(b) Compute $\int_0^{2\pi} f_n(x)\,dx$ (in terms of n).

(c) What happens to your answer in (b) as $n\to\infty$? Explain this result in terms of the graphs you found in (a).