Workshop 2

Sebtember 14, 2001

1. Let A be the region in the in the first quadrant of the x-y plane bounded by the y-axis, the line $y = 2$, and the curve $y = 2x^3$. We obtain solids of revolution S_1 and S_2 by revolving A about the lines $y = 5$ and $y = -3$, respectively. Both solids are (unusual) "doughnuts" which are 10 units across, whose hole is 6 units across, and whose thickness is 1 unit.

(a) Sketch these two solids.

(b) Which do you expect to have larger volume, S_1 or S_2? You should be able to determine this without any calculation. Explain your reasoning.

(c) Compute the volumes of S_1 and S_2 exactly and check that your conclusion in (b) is correct.

2. (a) Sketch the region bounded by the curve $y = \sqrt{x}$, the tangent line to this curve at $(1, 1)$, and the y-axis.

(b) Find the area of the region in (a) in two ways: first use x as the variable of integration; then use y as the variable of integration. Be careful in setting up the limits on the integrals.

(c) Find the volume generated when the region in (a) is rotated around the x axis.

3. (a) Sketch the region $0 \le y \le 1/x^2$, $2 \le x \le 8$.

(b) Find the number a such that the line $x = a$ divides the region in (a) into two parts of equal area.

(c) Find the number b such that the line $y = b$ divides the region in (a) into two parts of equal area.

4. Let R be the region lying in the first quadrant of the x-y plane and inside the circle $x^2 + y^2 = 1$. A certain solid has base R; cross-sections of the solid perpendicular to the y-axis are semicircles (with their diameters lying in R). Find the volume of the solid.