Workshop 1
September 7, 2001

1. (a) Find appropriate substitutions to evaluate the following indefinite integrals:

$$\text{(i)} \int x^2 \cos(1-x^3)\,dx; \qquad \text{(ii)} \int \frac{e^x}{1+e^x}\,dx; \qquad \text{(iii)} \int \frac{e^x}{1+e^{2x}}\,dx.$$

(b) Suppose f is continuous and $\int_0^8 f(u)\,du = 4$. Find $\int_0^2 x^2 f(x^3)\,dx$. (*Hint:* Try a substitution.)

(c) Use a substitution to evaluate $\displaystyle\int_0^1 x(1-x)^{1766}\,dx$. (It would be possible, but very tedious, to evaluate this by using the binomial theorem to expand $(1-x)^{1766}$.)

2. The function

$$C(x) = \int_0^x \cos(\pi t^2/2)\,dt$$

is used in the theory of diffraction of light waves.

(a) Find $C'(x)$ and plot its graph for $0 \le x \le 2$. On what intervals is $C(x)$ increasing?

(b) Find $C''(x)$ and plot its graph for $0 \le x \le 2$. On what intervals is $C(x)$ concave upwards?

(c) Use the graph in (a) to show that $1/2 < C(1) < 1$. Then use the `fnInt(` program on your calculator to find a more exact value for $C(1)$.

3. Let f be a continuous function defined everywhere.

(a) Show that for any $x > 0$, $\displaystyle\int_0^x tf(t^2)\,dt = \int_0^{x^2} f(u)\,du$.

(b) From (a) and the fundamental theorem of calculus find $\displaystyle\frac{d}{dx}\int_0^{x^2} f(u)\,du$.

(c) Let $\displaystyle F(y) = \int_0^y f(u)\,du$. Find $F'(y)$, then use the chain rule to find $\displaystyle\frac{d}{dx}F(x^2)$. You should find the same answer as you did in (b). Why?

4. Consider the quarter-disk in the first quadrant bounded by the axes and the curve $y = \sqrt{1-x^2}$. Divide it into 3 pieces as follows: pick any point (x, y) on the quarter circle and draw two lines from it, one to the origin, the other a vertical line down to the x-axis.

(a) Use known formulas for triangles and circular sectors to express the areas of two of the three pieces in terms of x. Express the area of the third piece as an integral.

(b) These three areas add up to $\pi/4$ (why?). Use that equation and the Fundamental Theorem to derive a formula for $\displaystyle\int \sqrt{1-x^2}\,dx$. Check the result by differentiating.