A set of solutions for these problems will be posted on the class web page on or before Sunday, December 8.

1. A region R in the first quadrant is bounded by the line $y = x$ and the parabola $y = -x^2 + 5x - 3$.
(a) Find the area of R.
(b) Find the volume of the solid obtained by rotating R around the y-axis.

2. Let R be the first quadrant region bounded by the curves $y = e^x$, $y = e^{-x}$ and the line $x = 1$.
(a) What is the area of R?
(b) What is the volume of the solid obtained by revolving R around the x-axis?
(c) What is the volume of the solid obtained by revolving R around the y-axis?

3. Determine the value of each of the definite integrals. Express the answer in terms of mathematical constants such as π or e, instead of numerical approximations.

(a) $\displaystyle\int_0^{\pi/8} \cos^4(x)\,dx$ (b) $\displaystyle\int_0^1 \tan^{-1}(x)\,dx$ (c) $\displaystyle\int_0^{\pi/2} \sin^3(x)\,dx$

4. Calculate the following indefinite integrals:

(a) $\displaystyle\int \frac{e^{2x}\,dx}{\sqrt{1+e^x}}$ (b) $\displaystyle\int \tan^2(x)\,dx$ (c) $\displaystyle\int \frac{dx}{(-3+4x-x^2)^{3/2}}$

(d) $\displaystyle\int x^3 \ln(x)\,dx$ (e) $\displaystyle\int \frac{x^5}{x^4-1}\,dx$ (f) $\displaystyle\int \frac{dx}{\sqrt{x^2+1}}$

5. Solve the differential equation $\dfrac{dy}{dx} = \cos^2(y)$ with the initial condition $y(0) = \dfrac{\pi}{4}$. In the answer, express y explicitly as a function of x.

6. Calculate each of the following improper integrals, or show that the integral is divergent:

(a) $\displaystyle\int_0^{\infty} \frac{\tan^{-1}x}{1+x^2}\,dx$ (b) $\displaystyle\int_0^{\infty} \frac{dx}{(x+1)(x+2)}$ (c) $\displaystyle\int_0^1 \frac{x\,dx}{(1-x^2)^{3/2}}$

7. Suppose that a is a constant with $a > 1$. Determine each of the sequential limits:

(a) $\displaystyle\lim_{n\to\infty} \frac{n^2}{a^n}$ (b) $\displaystyle\lim_{n\to\infty} \left(1+\frac{a}{n}\right)^n$ (c) $\displaystyle\lim_{n\to\infty} \frac{\ln(a^n+n^2)}{n}$ (d) $\displaystyle\lim_{n\to\infty} (\sqrt{n^2+an}-n)$

8. Test the following series for absolute convergence, conditional convergence, or divergence, explaining the test used and how it applies:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{\ln(n^2+1)}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^3}{(3n)!}$

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ (e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{2^n}$ (f) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$

9. A radioactive substance has half-life of 2 years. How long is required for 90% to decay, i.e., until only 10% of the original radioactive substance is left?

10. Determine the radius and interval of convergence of the following power series. In addition, determine those points at which the series is absolutely convergent.

$$\sum_{n=1}^{\infty} \frac{x^{3n}}{n^3 3^n}$$

11(a) Using $\ln(1+x) = \displaystyle\int_0^x \frac{dt}{1+t}$ and term-by-term integration, verify that the Maclaurin series of $\ln(1+x)$ is $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$, if $|x| < 1$.

11(b) Using #11(a) with $x = .5 = 1/2$, we approximate $\ln(1.5)$ with $\displaystyle\sum_{n=0}^{10} \frac{(-1)^n}{(n+1)2^{n+1}}$. Give an upper bound for the error in using this approximation.

12(a) Find the Maclaurin expansion of $F(x) = \displaystyle\int_0^x \tan^{-1}(t^2)\,dt$.

12(b) Use this series to find a numerical series that converges to the integral $\displaystyle\int_0^{1/2} \tan^{-1}(t^2)\,dt$.

13(a) Find the third Taylor polynomial, $T_3(x)$, of $f(x) = x^{1/4}$ centered at $a = 16$.
13(b) Use Taylor's inequality to give an upper bound for $|x^{1/4} - T_3(x)|$ when $16 \le x \le 18$.

14. The parametric curve

$$x(t) = \cos^3(t), \quad y(t) = \sin^3(t)$$

is often called the *astroid*.
(a) Sketch the astroid.
(b) Find $\dfrac{dy}{dx}$ as a function of t.
(c) Determine the equation of the tangent line to the curve at $t = \pi/6$.
(d) Find the arc length of the astroid for $0 \le t \le \pi/2$, and the total arc length of the astroid.

15. Sketch the polar curve $r = \theta^2$ for $0 \le \theta \le \pi$.
(b) Find the area bounded by this curve and the x-axis.
(b) Find the arc length of this curve.