## HW3

Graph Theory (Fall 2019) Rutgers University Swastik Kopparty

Due: Tuesday, October 15, 2019, in class

- 1. Problems 2.31, 2.32(a), the 2nd graph in 9.1, 9.2, 9.3(a), 9.5, 9.16, 9.19, 9.20, 10.1 (only G1), 10.3, 10.4, 10.11, 10.12, 10.17 (only G1).
- 2. Let G be a bipartite graph with  $10^7$  left vertices and 20 right vertices. Two vertices u, v are called twins if the set of neighbors of u equals the set of neighbors of v (triplets, quadruplets etc are defined similarly).

Show that G has twins.

## Bonus: Show that G has triplets. What about quadruplets, etc.?

- 3. Show that there exists a bipartite graph with  $10^5$  left vertices and 20 right vertices without any twins.
- 4. Show that any graph with n vertices and  $\delta(G) \ge n/2 + 1$  has a triangle.