Math - 251 Formula Sheet, Summer 2009

Trig identities: $\sin^2 t + \cos^2 t = 1$, $\cos^2 t = \frac{1}{2}(1 + \cos 2t)$, $\sin^2 t = \frac{1}{2}(1 - \cos 2t)$, $\sin 2t = 2 \sin t \cos t$

Second Derivatives Test: Suppose the second partial derivatives of $f = f(x, y)$ are continuous on a disk with center (x_0, y_0), and suppose that $f_x(x_0, y_0) = 0$ and $f_y(x_0, y_0) = 0$. Let $D = f_{xx} f_{yy} - f_{xy}^2$. If at (x_0, y_0):

(a) $D > 0$ and $f_{xx} > 0$, then $f(x_0, y_0)$ is a local minimum;

(b) $D > 0$ and $f_{xx} < 0$, then $f(x_0, y_0)$ is a local maximum;

(c) $D < 0$, then $f(x_0, y_0)$ is a saddle point;

(d) $D = 0$, then test is inconclusive.

Important formulas:

$$\oint_C F_1\,dx + F_2\,dy = \iint_D \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dA$$

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$$

$$\iiint_E \operatorname{div} \mathbf{F}\,dV = \oiint_S \mathbf{F} \cdot d\mathbf{S}$$

Additional formulas:

$$d\mathbf{S} = \mathbf{r}_u \times \mathbf{r}_v\,du\,dv$$

$$d\mathbf{S}_{\text{sphere}} = \mathbf{r}_\varphi \times \mathbf{r}_\theta\,d\varphi\,d\theta = a^2 \sin\varphi\,(\sin\varphi\cos\theta,\ \sin\varphi\sin\theta,\ \cos\varphi)$$