# Workshop 1, Math 152

**1.** Sketch the first quadrant region bounded by the parabola $y = x^2$, the tangent to the parabola at $(1, 1)$, and the $x$-axis, and calculate the area of this region.

**2.** Sketch the graph of $y_1 = \sin x$ and $y_2 = \cos x$ in the viewing window $[0, 2\pi] \times [-1, 1]$.
In this domain, find the area of the largest region that is bounded above by $y_1 = \sin x$ and below by $y_2 = \cos x$.

**3.** If $f$ is a continuous function, verify, using substitution, that $\displaystyle 2\int_0^1 f(2x+1)\,dx = \int_1^3 f(t)\,dt.$

**4.** Suppose that $0 < a < b$. Let $R$ be the region bounded by $y = 1/x$, $y = 0$, $x = a$, $x = b$.
(a) Find a vertical line $x = c$ that divides $R$ into two subregions of equal area. (Your answer should express $c$ as a function of $a$ and $b$.)
(b) Use a picture to explain the inequality: $\displaystyle \sqrt{ab} \le \frac{1}{2}(a+b)$.

**5.** Sketch the first quadrant regions:
(a) Bounded by $x^2 + y^2 = 1$ and $x + y = 1$
(b) Bounded by $\sqrt{x} + \sqrt{y} = 1$ and $x + y = 1$.
Which of the two regions has the larger area? Explain your answer .

**6.** Let $f(x) = x^3$.
(a) Find the equation of the tangent line to the graph $y = f(x) = x^3$ at the point $(1, 1)$.
(b) Find the finite area bounded by $y = x^3$ and the tangent line to $y = x^3$ at the point $(1, 1)$.

**7.** Calculate the integrals:

(a) $\displaystyle \int \frac{\tan(\sqrt{x})}{\sqrt{x}}\,dx$ (b) $\displaystyle \int \frac{\cos(\ln x)}{x}\,dx$ (c) $\displaystyle \int (2^x)^3(3^x)^2\,dx$

**8.** A positive function $f$ has the property that the area bounded by $y = f(x), y = 0, x = 0, x = u$ is given by the formula $\dfrac{u^2}{u^2+1}$ for all $u > 0$. What is the function $f$?

**9.** Sketch the region that is bounded above by the parabola $y = 1 - x^2$ and below by the $x$-axis.
(a) What is the area of this region?
(b) Find a horizontal line that divides the region into two subregions of equal area.