Math 151:19-21 Workshop 8 **Ian Coley** **29 October 2019**

Problem 1. Sketch the graph of a function $f(x)$ such that $f'(x) > 0$ when $x > 3$ and $f(x) < 0$ for $x < 3$

Problem 2. Sketch a graph of a function $g(x)$ satisfying the following: $g'(x) > 0$ for all x; $g''(x) < 0$ when $x < 0$; and $g''(x) > 0$ when $x > 0$.

Problem 3. Find the intervals on which the given is concave up or down, the points of inflection, the critical points, and the local minima and maxima for the following functions:

(a) $f(x) = \ln(x^2 + 2x + 5)$

(b) $g(\theta) = \theta + \sin\theta$ on $[0, 2\pi]$

Problem 4. Find conditions on b, c so that $f(x) = x^3 + bx + c$ is always increasing on $(-\infty, \infty)$.

Problem 5. Show that $f(x) = x^3 + ax^2 + bx + c$ is increasing on $(-\infty, \infty)$ as long as $b > a^2/3$.

Problem 6. We know that if $x = c$ is a critical point and $f''(c) = 0$, then the second derivative test is inconclusive. But if $f'(c) = 0$ and $f(c)$ is neither a local minimum nor a local maximum, must $x = c$ be a point of inflection? For "reasonable" functions, it does; but let's look at what happens for an "unreasonable" one: let

$$f(x) = \begin{cases} x^2 \sin\left(\dfrac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

(a) Use the limit definition of the derivative to prove that $f'(0)$ exists and equals 0. This means in particular that $f(x)$ is continuous at 0.

(b) Show that $f(0)$ is neither a local minimum nor local maximum. Hint: what is the sign of $f\left(\frac{2}{n\pi}\right)$ and $f\left(-\frac{2}{n\pi}\right)$ (in terms of n)? Also use the fact that in any small open interval around $x = 0$, we can always find some $\pm\frac{2}{n\pi}$ by making n big enough.

(c) Show that $f'(x)$ changes sign infinitely often near $x = 0$, so that $(0, 0)$ cannot be a point of inflection.