Math 151:19-21 Workshop 7 **Ian Coley** **22 October 2019**

Problem 1. As promised in lecture, compute the derivative of $f(x) = x^{x^x}$.

Problem 2. Show that for any real number k, $(1 + \Delta x)^k \approx 1 + k \cdot \Delta x$ for small Δx. Then estimate $(1.02)^7$ and $(1.02)^{-3}$. Hint: you'll want to do the linear approximation to a particular function at $a = 1$.

Problem 3. Consider the function $f(x) = \frac{1}{x}$. Compute $f'(x)$ and $f''(x)$. Then compute the approximate value of $f(1.1)$ (using the linear approximation) and give an upper bound for the error of this estimate.

Problem 4. Newton's method can be used to compute reciprocals without performing division. Let $c > 0$ and let $f(x) = x^{-1} - c$.

(a) What does a root of $f(x)$ correspond to?
(b) Show that the main step of Newton's method has the formula $x - f(x)/f'(x) = 2x - cx^2$.
(c) Estimate the reciprocal of 10.5 using the (good) guess $x_0 = 0.1$ and using the (bad) guess $x_0 = 0.5$. You only need to do Newton's method once.
(d) Using the relevant graph, explain why $x_0 = 0.1$ will tend towards the correct answer, but $x_0 = 0.5$ will not.

Problem 5. The extreme value theorem only applies to *closed* intervals. Sketch a graph of a continuous function $f(x)$ on $(0, 5)$ in each of the following cases:

(a) $f(x)$ has an absolute maximum but no absolute minimum
(b) $f(x)$ has an absolute maximum and an absolute minimum
(c) $f(x)$ has neither an absolute maximum nor absolute minimum

Then sketch a graph of a *discontinuous* function on $[0, 4]$ that has an absolute minimum but no absolute maximum. Hint: what sort of discontinuities can the function have?