Math 151:19-21 Workshop 4 **Ian Coley** **1 October 2019**

Problem 1. Suppose that $f(x)$ and $g(x)$ are differentiable functions, and we know the following:

$$f(2) = 2, \quad g(2) = -3, \quad f'(2) = 1, \quad g'(2) = 5$$

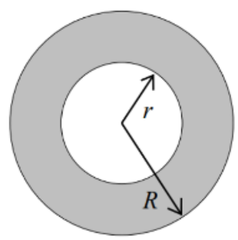
Compute the following:

(a) If $F(x) = \dfrac{f(x)}{g(x)}$, what is $F'(2)$?

(b) If $G(x) = x^3 \cdot f(x) - 7 \cdot g(x)$, what is $G'(2)$?

(c) If $H(x) = \dfrac{x^2 + e^x}{g(x)}$, what are $H(2)$ and $H'(2)$? Note: you should solve in terms of e.

Problem 2. Suppose that we have an annulus (or ring) that is composed of an outer circle with an inner circle removed:



We call r the *inner radius* and R the outer radius.

(a) If $r = 2$ and $R = 5$, what is the area of the annulus?

(b) Suppose that the inner radius is always $r = 2$, but the outer radius varies. Write a formula $A(R)$ for the area and compute $\dfrac{dA}{dR}$.

(c) Repeat part (b), but assuming that the inner radius is always half of the outer radius.

Problem 3. Students were asked the following: find all points where the graph of the function

$$f(x) = (x^2 - 1)(x^3 - 9x^2 + 24x + 92)$$

has a horizontal tangent line. Explain why the three solutions below are wrong, then give the correct strategy. Note: you'll likely need a calculator to solve this problem completely, so the strategy is enough.

(a) Horizontal tangent line means that we need to solve $f(x) = 0$. This means that $x^2 - 1 = 0$ or $x^3 - 9x^2 + 24x + 92 = 0$. Solving for x, we get $x = -1, 1$ for the first equation and $x = -2$ for the second equation (which only has one root).

(b) Horizontal tangent line means that we need to find $f'(0)$. So first we take the derivative,

$$f'(x) = \frac{d}{dx}(x^2 - 1)(x^3 - 9x^2 + 24x + 92) = (2x)(3x^2 - 18x + 24) = 6x^3 - 36x + 48x$$

We therefore conclude that $f'(0) = 0$.

(c) Horizontal tangent line means that we need to solve $f'(x) = 0$. So first we take the derivative,

$$f'(x) = \frac{d}{dx}(x^2 - 1)(x^3 - 9x^2 + 24x + 92) = (2x)(3x^2 - 18x + 24) = 6x^3 - 36x + 48x$$

and then factor it, so we need to solve $6x(x^2 - 6x + 8) = 0$. We can solve this to find the solutions are $x = 0, 2, 4$.