Problem 1. Consider the function $f(x) = \dfrac{2x-3}{\sqrt{2x^2-7x-15}}$.

(a) Find the domain and express it in interval notation.
(b) The domain is split into two disjoint pieces. Prove that $f(x) < 0$ on one of the pieces and $f(x) > 0$ on the other.
(c) This function has two horizontal asymptotes: one to the left at $y = -\sqrt{2}$ (approaching from below) and one to the right at $y = \sqrt{2}$ (approaching from above). Where are the vertical asymptotes?
(d) Can you give a sketch of this graph given this information? Can you determine if $f(x)$ is invertible?

Problem 2. A common use of exponentials is continuously compounded interest, which has the formula

$$P(t) = Ce^{rt}$$

where C is the initial investment, r is the interest rate, and t is the time (in years).

(a) Suppose that you invest 100 dollars into the bank at a rate of 5% per annum. How much money will you have in three years?
(b) How long will it take your 100 dollars to double to 200 dollars?
(c) If you instead invest 1000 dollars, how long will it take your money to double? Is your answer surprising?

Problem 3. A more interesting use of exponentials is to express complex numbers, i.e. expressions of the form $a + bi$ with $a, b \in \mathbb{R}$ where $i^2 = -1$ is the "imaginary" number. There's a famous formula of Euler: for any angle θ,

$$e^{i\theta} = \cos\theta + i\sin\theta.$$

Use this definition to prove the angle addition formulas:

$$\cos(x+y) = \cos x\cos y - \sin x\sin y, \quad \sin(x+y) = \sin x\cos y + \cos x\sin y.$$

Here's the idea: start with $e^{i(x+y)}$ and expand it using Euler's formula. Then remember that $e^{i(x+y)} = e^{ix} \cdot e^{iy}$ and do the same thing. Look at the two complex numbers you have: the real parts are the same, and the imaginary parts are the same. This shows that the angle angle formulas (and the double angle ones) are a secret type of "trig identity".

Problem 4. Let t be time in hours. Suppose that Dale got on the highway at $t = 0$. After two hours, he was 96 miles down the highway, when his tire pops and he pulls over. After another hour he was still on the side of the road, waiting for a two truck. What was Dale's average velocity over each of the time intervals:

(a) $t = 0$ to $t = 2$
(b) $t = 0$ to $t = 3$
(c) $t = 2$ to $t = 3$

What is the maximum (instantaneous) speed that Dale could have gone during his drive?