MATH 151: WORKSHOP 1

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Problem 1. A student was faced by the problem "Solve $|3x - 5| \geq 2$" and wrote

$$|3x - 5| = |3x + (-5)| = |3x| + | - 5| = |3x| + 5$$

and then concluded that $|3x - 5| \geq 2$ if and only if $|3x| \geq -3$, so this is true for all real numbers. What went wrong? What is the correct answer?

Problem 2. Assume that $f(x)$ and $g(x)$ are both odd functions defined on all real numbers. Consider the new functions $h(x) = f(x) \cdot g(x)$ and $k(x) = f(g(x))$. Prove that $h(x)$ is even and $k(x)$ is odd. Note: you can't prove this using graphs or examples!

Problem 3. Consider the functions $f(x) = \sqrt{x - 1}$ and $g(x) = \dfrac{1}{3x + 2}$.

(a) What is the domain of $f(x)$? Of $g(x)$?
(b) What is the equation for $(f \circ g)(x)$? What is its domain?
(c) What is the equation for $(g \circ f)(x)$? What is its domain?

Problem 4. Answer the following trigonometric problems:

(a) Compute $\sin(-7\pi/6)$ by comparing it to a "key" trigonometric value that you've memorised.
(b) Compute $\cos(\pi/12)$ using the double-angle identity and the difference-of-angles identity,
(c) If $\sin\theta = -\sqrt{3}/2$, what are the possible values for θ?
(d) If $\sin\theta = 2/7$, what are the possible values for $\tan\theta$?

Problem 5. Let $\theta = \cos^{-1}(-3/4)$.

(a) What quadrant is θ in?
(b) Using that information, draw a right triangle in which θ appears.
(c) Using that triangle, compute $\sin(\cos^{-1}(-3/4))$ with the appropriate sign.

Problem 6. Using the strategy in the previous problem, compute $\cos(\tan^{-1}(x))$.

Problem 7. Explain how the horizontal line test proves whether a function is injective or not. You may use graphs to demonstrate your argument, but make sure you use some words.