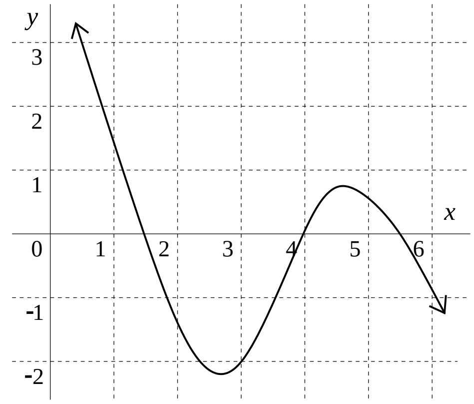
Problem statement

The graph of $y = f'(x)$, the *derivative* of the function $f(x)$, is shown to the right.



Graph of $f'(x)$, the *derivative* of $f(x)$

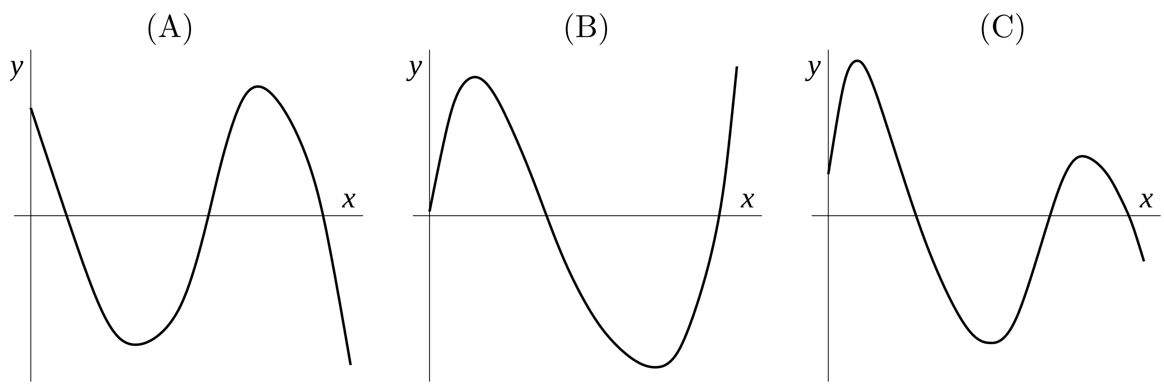
a) Use information from the graph of $f'(x)$ to find (as well as possible) the x where the *maximum value* of $f(x)$ in the interval $1 \le x \le 3$ must occur. Explain using calculus why your answer is correct (that is, why the value of $f(x)$ for the x you select is larger than $f(x)$ at *any* other number in the interval).

b) Suppose that $f(3) = 5$. Use information from the graph and the tangent line approximation for $f(x)$ to find an approximate value of $f(3.04)$. Explain using calculus and information from the graph why your approximate value for $f(3.04)$ is greater than or less than the exact value of $f(3.04)$.

Problem statement a) Suppose you know that $f'(x) = (x-1)(x-2)^2(x-3)^3(x-4)^4(x-5)^5$. What are the critical points of f? Which of them are local extrema, and what kind of local extrema are they?

b) Suppose you know that $g'(x) = x(x-1)^{2/3}(x-2)^{3/5}(x-3)^{4/7}$. What are the critical points of g? Which of them are local extrema, and what kind of local extrema are they?

Problem statement Below are the graphs of three functions $y = f(x)$. In just one of the graphs, it is true for all x that $\dfrac{d^3y}{dx^3} > 0$. Which is the graph? Explain why the other two graphs could not possibly satisfy the condition $\dfrac{d^3y}{dx^3} > 0$ for all x.



Problem statement Find the limits for the following indeterminate forms of the type "$\infty - \infty$".

a) $\lim_{x\to 0} \frac{1}{\sin x} - \frac{1}{x}$.

b) $\lim_{x\to 0} \frac{1}{x^2} - \frac{1}{x}$.

c) $\lim_{x\to 0} \frac{1+x}{x} - \frac{1-x}{x}$.