Problem statement A charged particle moves along the x-axis under the influence of an electric field. The field strength varies with time, and as a result the velocity of the particle is complicated. The position of the particle at time t is written as $x = x(t)$ and the velocity of the particle at time t is written as $v = v(t)$.
Suppose we know that $x(0) = 0$, and also that

$$v(t) = \begin{cases} 2t - 1, & \text{if } 0 \le t \le 1 \\ 4t - 3, & \text{if } 1 \le t \le 2 \\ 6t - 7, & \text{if } 2 \le t \le 3 \end{cases}.$$

What is $x(1)$? What is $x(2)$? What is $x(3)$? Sketch the graphs of $x = x(t)$ and $v = v(t)$.

Problem statement a) A car is traveling at 50 mi/h when the brakes are fully applied, producing a constant deceleration of 40 ft/s^2. What is the distance covered before the car comes to a stop?

b) A car braked with a constant deceleration of 40 ft/s^2 and produced skid marks measuring 160 feet before coming to a stop. How fast was the car traveling when the brakes were first applied?

Problem statement Define $S_6(n) = 1^6 + 2^6 + 3^6 + \cdots + n^6$ (in summation notation, $S_6(n) = \sum_{k=1}^{n} k^6$) then an explicit formula for $S_6(n)$ is known, and it is: $S_6(n) = \frac{1}{42}\left(6n^7 + 21n^6 + 21n^5 - 7n^3 + n\right)$. Similar formulas are known for other powers. These are sometimes called, collectively, *Faulhaber's formula*. Jacob Bernoulli also discovered these formulas but Faulhaber published earlier. It isn't even clear that the values of this formula are integers when n is an integer! **Assume that this formula is true.**

a) Check the formula for S_6 by evaluating it for $n = 4$. The answer should be the same as $1^6 + 2^6 + 3^6 + 4^6$.

b) Find some area and some approximating sum for this area which knowledge of this formula will allow you to evaluate exactly. Write the approximating sums, and evaluate the limit of these sums as $n \to \infty$ to compute the area.

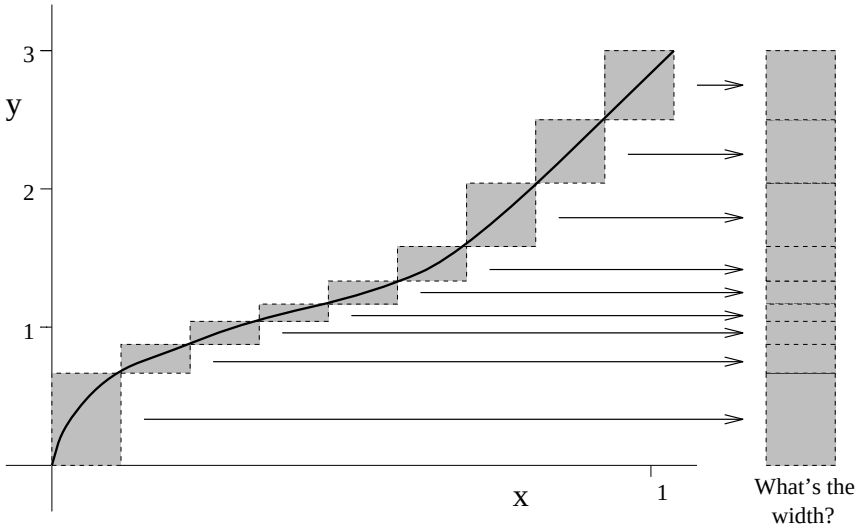
Problem statement Water is flowing through mighty Mill Brook. Two engineering students, Albert and Betty, are assigned the task of estimating the water flow through the brook between 8 AM and noon. They are provided with instruments which allow them to tell what the water flow is at any time in **gallons per minute**.

Problem statement Suppose $f(x) = \sqrt{3x + 6x^4}$.

a) Prove that f is increasing on the interval $[0, 1]$.

b) Write down a finite sum which will be within 10^{-10} of the true value of the area enclosed by the x-axis, $y = f(x)$, and $x = 1$. You are *not* asked to actually compute the sum, just describe it in any convenient fashion.

Hint Your reasoning and your explanation may be guided by the picture below. Note that the horizontal and vertical axes have different scales. The shaded rectangles represent the difference between right- and left-endpoint approximations.


3
y
2
1
x
1
What's the
width?