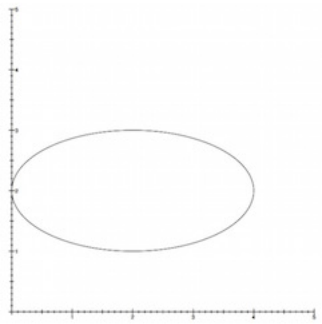
Problem 1 :

A cylinder vase having an ellipse base is placed in front of a wall as shown in the figure. Assume that x-axis represents the wall and the equation of the ellipse is given by $\frac{(x-2)^2}{4} + (y-2)^2 = 1$. If a candle is placed at point $(2, 4)$, what will be the length of the shadow on the wall?

Hint: Find the equations of the lines which are tangent to the ellipse and passing through $(2, 4)$.



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Problem 2 : Let $f(x)$ be given by

$$f(x) = \begin{cases} 3x, & \text{if } x \leq 0 \\ kx + 2kx^2, & \text{if } x > 0 \end{cases},$$

where k is some (unspecified) real number.

(a) For what values of k (if any) is the function $f(x)$ 1) continuous? 2) differentiable?

(b) Choose some k such that $f(x)$ is differentiable. Sketch the graphs of $f(x), f'(x)$ for this value of k.

(c) Does there exist a value of k such that $f'(x)$ is differentiable everywhere? If so, find this value of k, and if not, explain why not.

(d) Describe the function $f'''(x)$. Does it depend on k? What about the fourth or fifth derivatives of f?

Problem 3 An unidentified object moves along the s-axis, with displacement $s = s(t)$ (meters), velocity $v = v(t)$ (m/sec) and acceleration $a = a(t)$ (m/sec^2). It so happens that the velocity and displacement are related by the equation $v = \sqrt{8s + 16}$. Moreover, at the instant $t = 0$, the object is observed at $s = 6$.
a) Show that a is constant, and find its value.
b) Graph v as a function of s.
c) Graph v as a function of t.

Problem 4 An object is moving along the parabola $y = 3x^2$.
a) When it passes through the point $(2, 12)$, its "horizontal" velocity is $\frac{dx}{dt} = 3$. What is its "vertical" velocity at that instant?
b) If it travels in such a way that $\frac{dx}{dt} = 3$ for all t, then what happens to $\frac{dy}{dt}$ as $t \to +\infty$?
c) If, however, it travels in such a way that $\frac{dy}{dt}$ remains constant, then what happens to $\frac{dx}{dt}$ as $t \to +\infty$?